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Contributors to This Issue

Dr. J. C. Hunsaker, who has acted as Editor of this issue of the Journal of the Aeronautical Sciences is head of the Department of Mechanical Engineering at the Massachusetts Institute of Technology and Professor in charge of the course in Aeronautical Engineering. He consequently has found his task light because of the assistance of members of the Aeronautical Staff at M. I. T.

Dr. Th. von Karman is as international as science itself. The son of a Hungarian Professor of Theology, he was trained as an engineer at Budapest, as a scientist at Göttingen and Berlin, and has taught in Germany and Japan. He is Director of the Daniel Guggenheim Graduate School of Aeronautics at California Institute Technology. His contributions to the modern of skin friction and turbulence have been notable.

Dr. Richard H. Smith is Professor of Aeronautical Engineering at the Massachusetts Institute Technology. He obtained his doctorate at Hopkins University and training in aerodynamics by many years service as assistant to A. F. Zahm at the Navy's wind tunnel laboratory in Washington. Mr. J. Van H. Ve, who collaborates with Dr. Smith in ue, is a Research Associate at Massachusetts Institute of Technology who has been especially concerned with the development of precision apparatus.

Dr. L. Prandtl of Göttingen, the virtual father of modern aerodynamical theory, has developed a number of distinguished scientists from among his students and collaborators. Dr. Max M. Munk is a product of the Prandtl laboratory who came to this country after the war to engage in aerodynamic research at Langley Field. He designed the first compressed air wind tunnel there and made important contributions to applied hydrodynamics.

Prof. C. G. Rossby was born in Stockholm and graduated from its famous University. He was brought from the Norwegian Meteorological Service to this country by the Daniel Guggenheim Fund in 1927 to develop for American application the Norwegian methods of air mass analysis developed by the research staff of the Geophysical Institute of Bergen. Professor Rossby is a theoretical meteorologist and an ardent exponent of the application of the laws of thermodynamics and aerodynamics in weather analysis and forecasting.

Mr. A. Fage is an aerodynamical pioneer, having been engaged continuously on wind tunnel research since 1912 at Teddington in England. The editor of this issue learned some aerodynamic A.B.C.'s from Mr. Fage in the summer of 1913, when the science was young and its proponents few, but enthusiastic. Mr. Fage is a graduate of the Royal College of Science and a Fellow of the Royal Aeronautical Society.

Dr. A. J. Sutton Pippard is an expert in the theory of structures and teaches at Bristol University in England. During the War he was in charge of stress analysis work for the Air Ministry and since then has served on many boards to consider air-worthiness; in particular the air-worthiness of the airships R-100 and R-101. He is a Fellow of the Royal Aeronautical Society.

Mr. John Stack is an aerodynamical engineer engaged in wind tunnel research at Langley Field for the National Advisory Committee for Aeronautics.

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Turbulence and Skin Friction

TH. VON KÁRMÁN, *California Institute of Technology*

(Received November 3, 1933)

1. HISTORICAL DATA

THE first tests known on skin friction of flat surfaces in fluids are those carried out by M. G. Beaufoy[1] in 1793. About eighty years later W. Froude conducted a series of systematic experiments, investigating the influence of the velocity and the dimensions of the plate, especially that of the length in the direction of the motion, on the skin friction. In 1883 O. Reynolds published his fundamental investigations on the similarity of fluid motion and introduced the dimensionless parameter known as Reynolds number (referred to hereinafter as R.N.). In 1908 Lord Rayleigh pointed out that for geometrically similar arrangements, the skin friction coefficient, i.e., the friction per unit area divided by the head, must be a function of R.N. Succeeding investigations of Zahm, Wieselsberger, Gibson, with air, of the British Froude National Tank, Gebers, Kempf and the Washington Navy Yard with water, were evaluated using the R.N. as parameter.

Poiseuille gave in 1840 the theory of skin friction for the case of laminar flow in pipes. In 1850 G. Stokes published his calculations on the resistance of bodies in pure viscous flow, i.e., neglecting the inertia forces in the fluid. Prandtl and Blasius gave the mathematical theory of the laminar skin friction of plates in a free stream in 1904 and 1907. The motion of the boundary layer introduced by Prandtl proved itself useful through the further developments of the theory both for laminar and turbulent flow. The first semi-empirical theory of turbulent skin friction was given in 1921 by Prandtl and by the present author. The recent development of the theory presented in this paper started with the author's publications in 1930.

The scientific and technical literature on skin friction is very elaborate. An excellent bibliography, extending to 1930, has been compiled by A. F. Zahm and C. A. Ross.[2] Additional references are given at the end of this paper.

2. SKIN FRICTION AND MOMENTUM

The modern theory of skin friction is based on the theorem that friction between a fluid and a solid is accompanied by an equivalent change of the momentum carried by the fluid. Let us consider the case of a fluid moving along a flat plate and compare the momentum carried by the fluid through a cross section upstream from the leading edge of the plate and through a cross section at the distance x downstream from the leading edge. We assume that the velocity is uniform over the first cross section and has

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[2] Tentative bibliography on skin friction and boundary flow. Library of Congress, December, 1930.

the value U. The velocity normal to the second cross section at an arbitrary point therein shall be denoted by u. The fluid mass passing in unit time through an element dS of the second cross section is evidently $\rho u dS$, where ρ denotes the density of the fluid; this fluid mass had the momentum U per unit mass passing through the first cross section and has the momentum u per unit mass passing through the second cross section. Hence the loss of momentum of the fluid mass considered amounts to $\rho u(U-u)dS$ and the total loss of momentum of the fluid in unit time to $\int \rho u(U-u)dS$, the integral being taken over the cross section downstream. This amount is equal to the total frictional force acting on the portion of the plate extending from the leading edge to the distance x.

Let us consider a plate of the width b and introduce the coordinates x parallel to the direction of the main flow, y perpendicular to the flow and the plate with origin at the leading edge

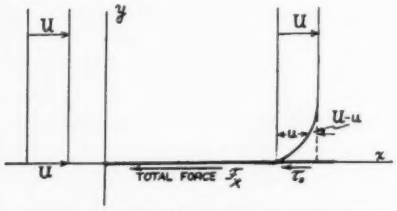

FIG. 1. Skin friction and boundary layer.

(Fig. 1). Then the friction F_x acting on the plate between $x=0$ (leading edge) and an arbitrary value of x will be

$$F_x = b\int_0^\infty \rho u(U-u)dy. \qquad (1)$$

The friction acting on a strip bdx is obviously equal to dF; on the other hand, the same force can be expressed by the *local friction* per *unit area* or *shearing stress* at the wall τ_0 in the form $b\tau_0 dx$. Hence we obtain

$$\tau_0 = \frac{1}{b}\frac{dF}{dx} = \frac{d}{dx}\left[\int_0^\infty \rho u(U-u)dy\right]. \qquad (2)$$

In all practical cases it is sufficiently exact to extend the integral from $y=0$ to a finite value $y=\delta$; we choose δ in such a way that the velocity defect $U-u$ is very small for values of y superior to δ. As a matter of fact, the influence of the friction is restricted to a narrow region along the plate; we call this region the "boundary layer" and δ "the thickness of the boundary layer." The choice of δ is, of course, somewhat arbitrary, but as it will be seen, the results concerning the laws and the values of the skin friction are independent of the definition of δ.

The consideration sketched above replaces the problem of the skin friction by the problem of the velocity distribution in the boundary layer. As a matter of fact, the friction itself can be determined as well by direct measurement as by measurement of the velocity distribution across the boundary layer.

3. THE THEORY OF LAMINAR SKIN FRICTION

In the case of laminar motion, the velocity distribution can be computed theoretically by using the hydrodynamic equations in their somewhat simplified form due to Prandtl (1904). The main result of this theory can be understood by using very little calculation if we assume similar velocity distributions over all cross sections of the boundary layer. Let us write, accordingly, $u = U\cdot f(y/\delta) = Uf(\eta)$ where $\eta = y/\delta$ is the "distance ratio" referred to the thickness δ of the boundary layer. The function $f(\eta)$ equals zero for $y=\eta=0$ and is approximately equal to one for $y=\delta$ or $\eta=1$. The "momentum integral" $\int_0^\infty \rho u(U-u)dy$ will be replaced by $\rho U^2\delta\int_0^1 f\cdot(1-f)d\eta$.

The integral $\int_0^1 f\cdot(1-f)d\eta$ is a pure number which shall be denoted by α. On the other hand, the friction at the wall τ_0, according to the general law of viscosity, equals the product of the viscosity coefficient μ and the slope of velocity du/dy at the wall. Hence

$$\tau_0 = \mu(du/dy)_{y=0} = \mu(U/\delta)(df/d\eta)_{\eta=0}. \qquad (3)$$

The value $(df/d\eta)_{\eta=0}$ is also a pure number, we call it β. Introducing the value of τ_0 and that of the momentum integral in Eq. (2) we obtain

$$\mu\cdot\frac{U}{\delta}\cdot\beta = \frac{d}{dx}(\alpha\rho U^2\delta) \quad \text{or} \quad \delta\frac{d\delta}{dx} = \frac{\mu}{\rho}\cdot\frac{1}{U}\cdot\frac{\beta}{\alpha}.$$

By integration, $\frac{1}{2}\delta^2 = \frac{\mu\beta}{\rho U\alpha}x$

and

$$\delta = [(2\beta/\alpha)(\mu x/\rho U)]^{\frac{1}{2}}. \tag{4}$$

Putting this value of δ in Eq. (3) we obtain the value of the local friction

$$\tau_0 = \mu\beta(U/\delta) = (\tfrac{1}{2}\alpha\beta)^{\frac{1}{2}} \cdot (\rho\mu U^3/x)^{\frac{1}{2}} \tag{5}$$

and by integration, the value of the total friction

$$F_x = (2\alpha\beta)^{\frac{1}{2}} \cdot (\rho\mu x U^3)^{\frac{1}{2}} b. \tag{6}$$

We introduce the coefficient of the local friction

$$c_f = \tau_0/\tfrac{1}{2}\rho U^2 \tag{7}$$

and that of the total (or mean) friction

$$C_f = F_x/\tfrac{1}{2} \cdot \rho \cdot U^2 \cdot b \cdot x. \tag{8}$$

The mathematical theory gives for $(2\alpha\beta)^{\frac{1}{2}}$ the numerical value 0.664. Hence with $\mu/\rho = \nu$

$$c_f = 0.664/(Ux/\nu)^{\frac{1}{2}} \tag{9}$$

and

$$C_f = 1.328/(Ux/\nu)^{\frac{1}{2}}. \tag{10}$$

We call Ux/ν the R.N. of the plate R_x.

Summarizing the results given by the equations from (4) to (10), we obtain the following statements (cf. Fig. 2):

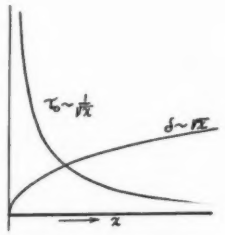

FIG. 2. Distribution of laminar skin friction and thickness of laminar boundary layer.

(a) The increase of the thickness of the laminar boundary layer δ is proportional to the square root of the distance from the leading edge and inversely proportional to the square root of the relative velocity between plate and fluid.

(b) The local friction per unit area is proportional to the 3/2 power of the velocity, inversely proportional to the square root of the distance from the leading edge and proportional to the geometrical mean value between viscosity and density.

(c) The friction coefficients both for the local and the mean friction are inversely proportional to the square root of the R.N.

4. Power Laws for Turbulent Skin Friction

The method of the last section can be extended to the case of the turbulent boundary layer without investigating the actual mechanism of turbulent motion and by using merely the method of dimensional analysis.

We assume, as before, that the velocity distribution is given by a function of the form $u = Uf(y/\delta)$, but for the wall friction we use an empirical law of the form $\tau_0 = \frac{1}{2}\rho U^2 g(U\delta/\nu)$. Using a "power law" as a convenient interpolation formula, we put

$$\tau_0 = \tfrac{1}{2}\rho U^2 \cdot \text{const.}/(U\delta/\nu)^m. \tag{11}$$

Evidently the momentum integral $\int_0^\delta \rho u(U-u)dy$ keeps the form $\alpha\rho U^2\delta$, where α is a numerical value, which depends on the shape of the velocity distribution in the boundary layer. Hence we obtain from the fundamental Eq. (2)

$$\alpha\rho U^2(d\delta/dx) = \tfrac{1}{2}\rho U^2 \cdot \text{const.}/(U\delta/\nu)^m \tag{12}$$

and

$$\delta^m(d\delta/dx) = \text{const.} \cdot (\nu/U)^m. \tag{12a}$$

Integrating (12a) and assuming $\delta = 0$ for $x = 0$, we obtain

$$\delta^{m+1}/(m+1) = \text{const.} \cdot (\nu/U)^m \cdot x$$

or

$$\delta = \text{const.} \cdot x \cdot (\nu/Ux)^{m/(m+1)}. \tag{13}$$

This equation determines the "development of the boundary layer" along the plate, if the "elementary friction law" (11) is known.

Introducing the value of δ in Eq. (11) for τ_0, we write

$$\tau_0 = \tfrac{1}{2}\rho U^2 \cdot \text{const.}/(Ux/\nu)^{m/(m+1)}$$

or

$$c_f = \text{const.}/(Ux/\nu)^{m/(m+1)}. \tag{14}$$

We started with Eq. (11) which expressed the relation between the friction and the R.N. of the

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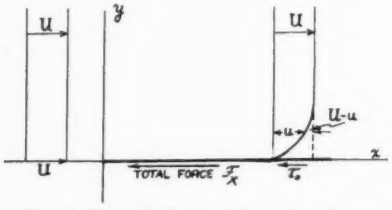

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The consideration sketched above replaces the problem of the skin friction by the problem of the velocity distribution in the boundary layer. As a matter of fact, the friction itself can be determined as well by direct measurement as by measurement of the velocity distribution across the boundary layer.

3. THE THEORY OF LAMINAR SKIN FRICTION

In the case of laminar motion, the velocity distribution can be computed theoretically by using the hydrodynamic equations in their somewhat simplified form due to Prandtl (1904). The main result of this theory can be understood by using very little calculation if we assume similar velocity distributions over all cross sections of the boundary layer. Let us write, accordingly, $u = U\cdot f(y/\delta) = Uf(\eta)$ where $\eta = y/\delta$ is the "distance ratio" referred to the thickness δ of the boundary layer. The function $f(\eta)$ equals zero for $y=\eta=0$ and is approximately equal to one for $y=\delta$ or $\eta=1$. The "momentum integral" $\int_0^\infty \rho u(U-u)dy$ will be replaced by $\rho U^2\delta\int_0^1 f\cdot(1-f)d\eta$.

The integral $\int_0^1 f\cdot(1-f)d\eta$ is a pure number which shall be denoted by α. On the other hand, the friction at the wall τ_0, according to the general law of viscosity, equals the product of the viscosity coefficient μ and the slope of velocity du/dy at the wall. Hence

$$\tau_0 = \mu(du/dy)_{y=0} = \mu(U/\delta)(df/d\eta)_{\eta=0}. \qquad (3)$$

The value $(df/d\eta)_{\eta=0}$ is also a pure number, we call it β. Introducing the value of τ_0 and that of the momentum integral in Eq. (2) we obtain

$$\mu\cdot\frac{U}{\delta}\cdot\beta = \frac{d}{dx}(\alpha\rho U^2\delta) \quad \text{or} \quad \delta\frac{d\delta}{dx} = \frac{\mu}{\rho}\cdot\frac{1}{U}\cdot\frac{\beta}{\alpha}.$$

By integration, $\frac{1}{2}\delta^2 = \frac{\mu\beta}{\rho U\alpha}x$

and

$$\delta = [(2\beta/\alpha)(\mu x/\rho U)]^{\frac{1}{2}}. \tag{4}$$

Putting this value of δ in Eq. (3) we obtain the value of the local friction

$$\tau_0 = \mu\beta(U/\delta) = (\tfrac{1}{2}\alpha\beta)^{\frac{1}{2}} \cdot (\rho\mu U^3/x)^{\frac{1}{2}} \tag{5}$$

and by integration, the value of the total friction

$$F_x = (2\alpha\beta)^{\frac{1}{2}} \cdot (\rho\mu x U^3)^{\frac{1}{2}} b. \tag{6}$$

We introduce the coefficient of the local friction

$$c_f = \tau_0 / \tfrac{1}{2}\rho U^2 \tag{7}$$

and that of the total (or mean) friction

$$C_f = F_x / \tfrac{1}{2} \cdot \rho \cdot U^2 \cdot b \cdot x. \tag{8}$$

The mathematical theory gives for $(2\alpha\beta)^{\frac{1}{2}}$ the numerical value 0.664. Hence with $\mu/\rho = \nu$

$$c_f = 0.664/(Ux/\nu)^{\frac{1}{2}} \tag{9}$$

and

$$C_f = 1.328/(Ux/\nu)^{\frac{1}{2}}. \tag{10}$$

We call Ux/ν the R.N. of the plate R_x.

Summarizing the results given by the equations from (4) to (10), we obtain the following statements (cf. Fig. 2):

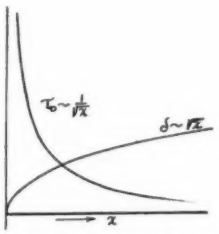

FIG. 2. Distribution of laminar skin friction and thickness of laminar boundary layer.

(a) The increase of the thickness of the laminar boundary layer δ is proportional to the square root of the distance from the leading edge and inversely proportional to the square root of the relative velocity between plate and fluid.

(b) The local friction per unit area is proportional to the 3/2 power of the velocity, inversely proportional to the square root of the distance from the leading edge and proportional to the geometrical mean value between viscosity and density.

(c) The friction coefficients both for the local and the mean friction are inversely proportional to the square root of the R.N.

4. Power Laws for Turbulent Skin Friction

The method of the last section can be extended to the case of the turbulent boundary layer without investigating the actual mechanism of turbulent motion and by using merely the method of dimensional analysis.

We assume, as before, that the velocity distribution is given by a function of the form $u = Uf(y/\delta)$, but for the wall friction we use an empirical law of the form $\tau_0 = \frac{1}{2}\rho U^2 g(U\delta/\nu)$. Using a "power law" as a convenient interpolation formula, we put

$$\tau_0 = \tfrac{1}{2}\rho U^2 \cdot \text{const.}/(U\delta/\nu)^m. \tag{11}$$

Evidently the momentum integral $\int_0^\delta \rho u(U-u)dy$ keeps the form $\alpha\rho U^2\delta$, where α is a numerical value, which depends on the shape of the velocity distribution in the boundary layer. Hence we obtain from the fundamental Eq. (2)

$$\alpha\rho U^2(d\delta/dx) = \tfrac{1}{2}\rho U^2 \cdot \text{const.}/(U\delta/\nu)^m \tag{12}$$

and

$$\delta^m(d\delta/dx) = \text{const.} \cdot (\nu/U)^m. \tag{12a}$$

Integrating (12a) and assuming $\delta = 0$ for $x = 0$, we obtain

$$\delta^{m+1}/(m+1) = \text{const.} \cdot (\nu/U)^m \cdot x$$

or

$$\delta = \text{const.} \cdot x \cdot (\nu/Ux)^{m/(m+1)}. \tag{13}$$

This equation determines the "development of the boundary layer" along the plate, if the "elementary friction law" (11) is known.

Introducing the value of δ in Eq. (11) for τ_0, we write

$$\tau_0 = \tfrac{1}{2}\rho U^2 \cdot \text{const.}/(Ux/\nu)^{m/(m+1)}$$

or

$$c_f = \text{const.}/(Ux/\nu)^{m/(m+1)}. \tag{14}$$

We started with Eq. (11) which expressed the relation between the friction and the R.N. of the

boundary layer. It is logical to assume that this relation is similar to that obtained for a channel or pipe with constant cross section. For laminar flow we have therefore, according to Poiseuille's law, $m=1$ and therefore $m/m+1=\frac{1}{2}$ so that Eq. (13) leads to the results of the last section. For turbulent flow tests on pressure drop in smooth pipes give in the range $3000<Ur/\nu<70{,}000$ (r = radius of circular pipe) good accordance with the empirical Eq. (11) choosing the exponent $m=\frac{1}{4}$. Using this exponent, Prandtl and the present author obtained from (14) the formula for the turbulent skin friction coefficient of a smooth plate:

$$c_f=\text{const.}/(Ux/\nu)^{1/5} \tag{15}$$

and with the value of the numerical constant deduced from the pipe tests

$$c_f=0.059/(Ux/\nu)^{1/5}.$$

It is easy to show that the coefficient of the mean friction is equal to $C_f=c_f\cdot(m+1)/m$ and therefore

$$C_f=0.074/(Ux/\nu)^{1/5}. \tag{16}$$

This formula checks the tests with smooth plates in the range extending to about $Ux/\nu\leqq 3\cdot 10^6$.

Extension of the tests to larger R.N. made it evident that the power formula with a constant exponent could not serve for a larger range of R.N., either in the case of the pipe or in the case of the plate. It was found that the exponent m decreases with increasing R.N. This proved the assumption of similar velocity distributions to be incorrect. In order to find a law valid in the whole range and especially valid for large R.N. allowing extrapolation to full scale conditions, a more systematic investigation on the nature of turbulent flow became inevitable. This new development of the theory achieved in the last few years is shortly reviewed in the following sections.

5. The Mechanism of Turbulent Flow

In laminar flow the interaction between adjacent fluid layers consists of so-called "molecular friction"; the shearing stress, i.e., the friction per unit area, is equal to the product of the rate of shear (slope of velocity in the case of parallel flow) and the viscosity coefficient. The latter is a characteristic physical parameter of the fluid, dependent chiefly on the temperature, practically independent of the velocity. In turbulent flow an additional interaction is due to the momentum transfer from layer to layer due to the velocity fluctuations. Let us consider a flow, consisting of parallel mean motion in the x-direction, the temporal mean value u of the velocity being a function of y only. O. Reynolds has shown that the momentum transport in unit time and through unit area due to the fluctuations represents an apparent shearing stress for the mean motion. This apparent shearing stress, or "turbulent friction," has the magnitude $\tau=-\rho\overline{u'v'}$ where u' and v' are the fluctuations of the velocity components in the x- and y-direction, and the dash denotes temporal mean value of the product. In fact the fluid volume passing through a unit area perpendicular to the y-direction is equal to v' and the excess of momentum in the direction of the mean flow carried by the fluid per unit volume is equal to $\rho u'$. The negative sign corresponds to the definition of positive friction in such a way that τ is considered as positive if the fluid layer at the distance y from the wall is accelerated by the outside flow; accordingly, the momentum transfer produces friction of positive amount if the fluid portions moving toward the wall (i.e., having negative v' components) carry positive excess of momentum.

In the case of velocity fluctuations with a random distribution of the u' and v' components, the product $u'v'$ has the mean value zero. The shearing stress is different from zero only in the case of a certain correlation between u' and v'. It is easy to see that a correlation can be expected in all cases in which the mean velocity u is variable with y. Let us assume for instance that $u(y)$ is increasing with y (Fig. 3). Then the

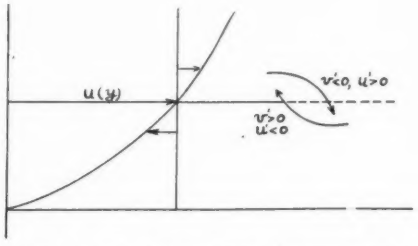

FIG. 3. Turbulent exchange of momentum.

fluid masses moving in the negative y-direction ($v' < 0$), i.e., coming from a region of higher velocity, probably have larger u-components than the masses with positive v' values, i.e., coming from a region of lower velocity. Now evidently larger u-velocities than the mean velocity at the place considered appear as positive, smaller u-velocities as negative u' fluctuations; hence negative v' values will be connected with positive u' values and *vice versa*. Therefore, we have to expect a negative value for the mean product $\overline{u'v'}$ and a positive value for the shearing stress. Fig. 4 shows schematically the random

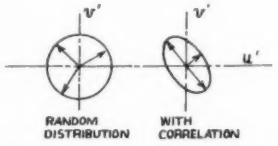

FIG. 4. Diagram of velocity fluctuations.

and correlated distribution of the velocity fluctuations.

The main characteristics of the turbulent flow at a certain point are the magnitude of the fluctuations u', v', w', and the correlations between them. The theory of correlations uses a certain ratio between mean values as "correlation factor." For instance, the correlation between the components u' and v' is measured by the ratio $\overline{u'v'}/(\overline{u'^2})^{\frac{1}{2}}(\overline{v'^2})^{\frac{1}{2}}$ where $(\overline{u'^2})^{\frac{1}{2}}$ and $(\overline{v'^2})^{\frac{1}{2}}$ are the square roots of the mean squares of the particular components. This correlation factor is zero for random distribution and is equal to unity if u' and v' are proportional. The ellipse shown in Fig. 4 becomes a circle in the first case and a straight line in the latter.

Considering the "flow pattern" in the neighborhood of a certain point, we have to introduce a further characteristic parameter of the turbulent flow: namely a length which is characteristic for the size of the region involved in the turbulent exchange, i.e., in the turbulent momentum transfer. The corresponding parameter in the molecular mechanism of friction is the mean free path. Prandtl uses for the length-parameter of the turbulent friction-mechanism the expression "mixing length." His particular picture of the turbulent exchange is based on the assumption that the fluid masses displaced perpendicular to the direction of mean flow carry their momentum invariably over a certain length perpendicular to the mean flow. The actual mechanism may be more complicated; in any case the conception of a characteristic length is necessary for the understanding of the fundamental laws of turbulent flow. Let us compare, for instance, the turbulent fluctuations in the boundary layer of an airfoil placed in the wind tunnel and those in the boundary layer produced by the natural wind on the ground. Mean velocities and velocity fluctuations may have the same order of magnitude; it is obvious that the length scale of the flow pattern in the two cases will be very different.

There is a further factor to be considered in connection with the characteristic length or with the length scale of the flow pattern, namely the time-factor or the time-scale of the fluctuations. "Large scale turbulence" will be connected in general with fluctuations of large period, and "small scale turbulence" with fast fluctuations.

The characteristic features of turbulent flow have been dealt with in some detail and from a general point of view, because the author feels that clear notions on this question are of importance for understanding the phenomena and for further research. Common language, even in technical papers, uses the term "turbulence" in a rather indefinite way, including in it for instance "vortex motion" in general. The scientific term "turbulence," as used in this paper, implies "irregular fluctuations," governed by laws of some statistical equilibrium. To be sure, turbulence may be originated by vortices as in the case represented in Fig. 5, showing the flow through a honeycomb; but we call such a motion "turbulent" only if the regular pattern, because of the intermingling of a great number of vortices, disappears, as happens farther downstream from the honeycomb. Similarly, vortices produced by obstacles at the ground contribute essentially to atmospheric turbulence, but also in this case we restrict the term turbulence to the statistical phenomenon of the mass exchange and keep this case apart from other conceptions, such as individual vortices (tornados, vortex sheets, etc.) or regular atmospheric waves.

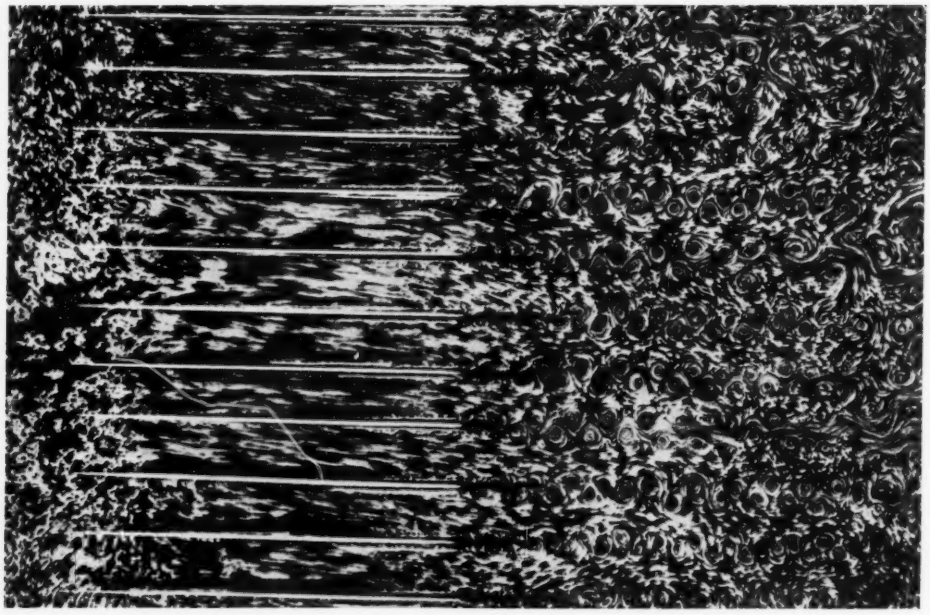

FIG. 5. Vortex rows produced by a honeycomb. (Photograph by Ahlborn.)

6. THE BASIC LAWS FOR THE MEAN VELOCITY DISTRIBUTION IN TURBULENT FLOW

Adding the momentum transfer by turbulent fluctuations to the laminar friction, O. Reynolds obtained the following expression for the total friction (shearing stress):

$$\tau = \mu \cdot du/dy - \rho\overline{u'v'}. \qquad (17)$$

Calculating τ from the measured values of the pressure drop in pipes or the skin friction of plates, the values $\mu(du/dy)$ from the measured mean velocity distribution, we find that in the practically important range of high R.N. the laminar friction is a negligible part of the total friction except in the immediate neighborhood of the wall. Hence we conclude that except for this region the turbulent exchange is practically independent of the viscosity of the fluid. This assumption is essentially substantiated by experimental facts concerning the velocity distribution in pipes and channels. Let us consider, for instance, turbulent flow between two parallel walls (rectangular pipe with great aspect ratio). The velocity in the center will be denoted by U, the velocity at a distance y from the wall by u, the distance between center and wall by r, the wall friction by τ_0. The difference $(U-u)$ will be called the "velocity defect." We find experimentally that for high R.N. the ratio $(U-u)/(\tau_0/\rho)^{\frac{1}{2}}$ is a general function of the ratio y/r or $(r-y)/r$, i.e., independent of R.N. and also independent of the roughness of the wall.

A simple interpretation of this result can be given by comparing cases in which both τ_0 and r have the same values, but the roughness of the wall is different (Fig. 6). The velocity defect $(U-u)$ is the relative velocity between the fluid moving in the center layer and the fluid moving in a layer at a distance $(r-y)$ from the center.

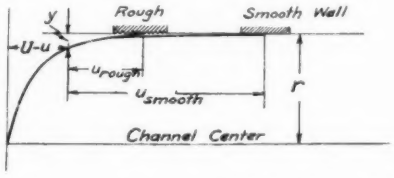

FIG. 6. Comparison of velocity distributions in a smooth and in a rough channel.

Considering the flow relative to the fluid moving in the center layer, the fact that $(U-u)$ is independent of the roughness of the wall and is proportional to $(\tau_0/\rho)^{\frac{1}{2}}$ means that the action of the wall friction is independent of the way the friction is produced: whether by slow displacement of a very rough wall or by fast displacement of a smooth wall. The mechanism of the turbulent exchange seems to be the same in both cases, except in the immediate neighborhood of the walls.

The same experimental result is true for the flow in a pipe with circular cross section. Denoting the radius of the pipe by r we obtain

$$(U-u)/(\tau_0/\rho)^{\frac{1}{2}}=f(y/r), \tag{18}$$

where $f(y/r)$ is a universal function.

The quantity $(\tau_0/\rho)^{\frac{1}{2}}$ has evidently the dimension of a velocity. In aerodynamics it is usual to characterize a velocity by the corresponding pressure head; in the inverse way we can characterize a certain value of the pressure p by the corresponding velocity $(2p/\rho)^{\frac{1}{2}}$. In a similar way we can assign the velocity $(2\tau_0/\rho)^{\frac{1}{2}}$ to the shearing stress τ_0. The pressure head corresponding to this velocity is numerically equal to the shearing stress. Instead of $(2\tau_0/\rho)^{\frac{1}{2}}$ we will use $(\tau_0/\rho)^{\frac{1}{2}}$. To be sure it would be more logical to use the former quantity, but $(\tau_0/\rho)^{\frac{1}{2}}$ has been used already in so many publications that a change would likely produce confusion. We call the quantity $(\tau_0/\rho)^{\frac{1}{2}}$ the *friction velocity*. The ratio y/r can be called the *relative distance from the center*.

Hence the statement expressed by Eq. (18) can be worded in the following way: The *ratio* $\left(\frac{\text{velocity defect}}{\text{friction velocity}}\right)$ *is a universal function of the relative distance from the center.*

Let us now consider the velocity distribution relative to the wall. At the wall itself the velocity is zero or equal to the velocity of the wall, if the wall is moving. Hence, at the wall the velocity components u' and v' vanish, and the momentum transfer is zero, so that the friction is merely laminar friction. Thus we obtain a "laminar sublayer" adjacent to the wall with gradual transition to the turbulent region. Therefore, the velocity distribution as function of the distance y will contain not only the wall friction τ_0 and the density ρ, but also the kinematic viscosity ν. Let us restrict the investigation to the case of perfectly smooth walls. In this case it can be expected that near the wall u is in fact fully determined by τ_0, y, ρ and ν. Dimensional analysis shows easily that the only possible combination is given by the formula

$$u=(\tau_0/\rho)^{\frac{1}{2}}g((\tau_0/\rho)^{\frac{1}{2}}y/\nu), \tag{19}$$

where g is another universal function.

The parameter $(\tau_0/\rho)^{\frac{1}{2}}y/\nu$, introduced first by L. Prandtl, is built up in the same way as the R.N.; it contains the friction velocity, the distance and the kinematic viscosity. We call it the *friction-distance parameter.*

Hence Eq. (19) says the *ratio* $\frac{\text{velocity}}{\text{friction velocity}}$ *near the wall is a universal function of the friction-distance parameter.*

7. Similarity of the Turbulent Flow Pattern

In order to determine the function $f(y/r)$ representing the velocity distribution over the cross section of a pipe or a channel, the author introduced the assumption that the flow pattern of the turbulent interchange in the neighborhood of different points is similar and only different as far as length and time scale are concerned. This assumption appears to be justified in all cases in which the characteristic length of the turbulent exchange is small in comparison with the dimensions of the cross section.

The following conclusions are deduced from the above-mentioned assumption:

(a) The velocity fluctuations are proportional to a characteristic length l of the flow pattern and to the slope of the mean velocity du/dy.

(b) The shearing stress τ, variable over the cross section, is at every point proportional to $\rho l^2(du/dy)^2$. Hence l is heretofore only a relative measure of the flow pattern; we can absorb the proportionality factor and define l by writing $\tau=\rho l^2(du/dy)^2$.

(c) The characteristic length is proportional to the ratio $|(du/dy)/(d^2u/dy^2)|$. We write $l=\kappa|(du/dy)/(d^2u/dy^2)|$, where κ is a numerical

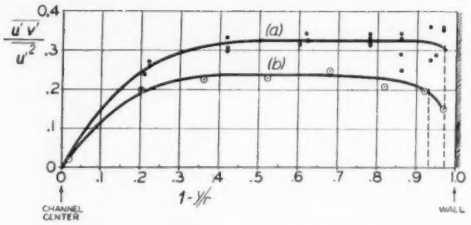

FIG. 7. Correlation between turbulent velocity fluctuations. (a) Wattendorf (C.I.T.). (b) Reichardt (Göttingen).

factor. It will be seen that the constant κ occurs in most of the formulae given in the later sections; it appears as a "universal constant" of the turbulent exchange.

The formula $\tau = \rho l^2 (du/dy)^2$ was first given by Prandtl in his theory of the mixing length, assuming a special kind of mechanism for the turbulent exchange of momentum.

The assumption of similar flow patterns is identical with the assumption of *constant correlation* between the components of the velocity fluctuations. There is not much experimental evidence on the correlation of the fluctuations, but in some cases the magnitude of the fluctuations has been measured. Recently F. Wattendorf of the California Institute of Technology measured the value of $\overline{u'^2}$ over the cross section of a rectangular channel with aspect ratio 1 : 18, using a hot wire anemometer suitable for fast fluctuations. In this case the value of $\overline{u'v'}$ could be deduced from the measurement of the pressure drop, assuming $\tau = -\rho \overline{u'v'}$ and taking into consideration that the shearing stress τ in such a channel is almost exactly proportional to the distance from the center. Thus the ratio $\overline{u'v'}/\overline{u'^2}$ could be calculated; the results are given in Fig. 7 (curve (*a*)). The points connected by the curve (*b*) correspond to similar measurements by Reichardt (Göttingen). The curves show the following results:

(a) The ratio $\overline{u'v'}/\overline{u'^2}$ is almost constant over the main part of the cross section, corresponding to constant correlation.

(b) For the center $\overline{u'v'}/\overline{u'^2} \to 0$; this is evident by symmetry considerations, since in the center no reason for correlation exists. Accordingly, in the center part the similarity assumption cannot be correct.

(c) The curve (*b*) drops near the wall. However, Reichardt's experiments are carried out at a relatively low R.N., so that the range of viscosity influence is comparatively large. The estimated range in which $\mu\, du/dy$ cannot be neglected in comparison with $-\rho \overline{u'v'}$ is indicated by the dotted lines for both cases. The way we can estimate the thickness of the range influenced by laminar friction is shown in the latter part of this section.

The assumption of similarity of the turbulent flow pattern gives the following formula for the velocity distribution:

$$\frac{U-u}{(\tau_0/\rho)^{\frac{1}{2}}} = -\frac{1}{\kappa}\left[\log\left(1-\left(1-\frac{y}{r}\right)^{\frac{1}{2}}\right)+\left(1-\frac{y}{r}\right)^{\frac{1}{2}}\right].$$

The formula checks the measured velocity distributions very well (Fig. 8).

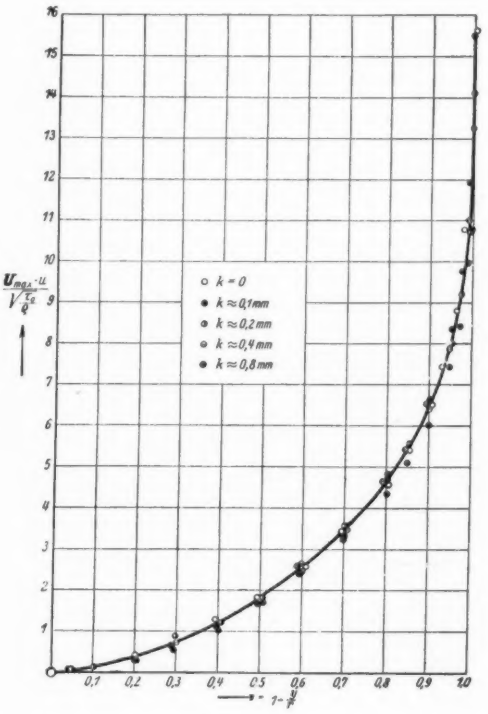

FIG. 8. Measured velocity distributions compared with the theoretical curve.

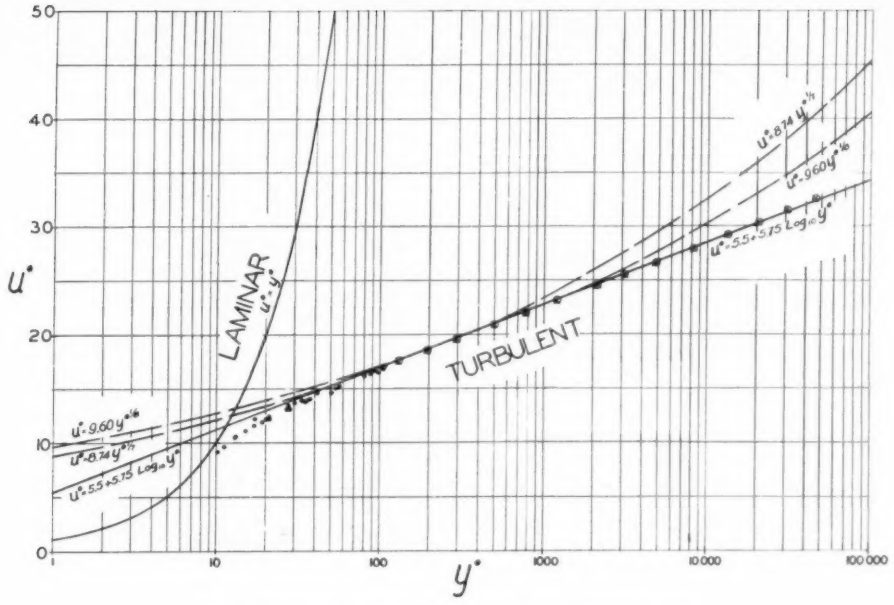

FIG. 9. Universal velocity distribution near a smooth wall.

For small values of y/r, we obtain

$$\left.\begin{aligned} U_{\max}-u &= -(1/\kappa)(\tau_0/\rho)^{\frac{1}{2}} \log (y/r) \\ \text{or} \qquad u &= \text{const.}+(1/\kappa)(\tau_0/\rho)^{\frac{1}{2}} \log y \end{aligned}\right\}. \quad (20)$$

This result can be seen directly in the following way:

Let us consider the idealized case that a constant shearing stress is transferred in a parallel flow along the wall. In this case an exact similarity of the flow pattern can be expected; in fact looking at the flow from the point of view of two observers travelling with the fluid with a velocity equal to the mean velocity of the fluid, at two different distances y_1 and y_2 from the wall, nothing can be different in the flow pictures, conceived by the two observers, except the length and scale of the flow pattern. Thus the characteristic length of the flow pattern is proportional to y. The distribution of the mean velocity is, according to Taylor's theorem, given by

$$u(y+\Delta y) = u(y)+u'\Delta y+\tfrac{1}{2}u''\Delta y^2+\cdots.$$

It is evident that the similarity of the flow pattern involves $u'/u'' = u''/u''' = \cdots \doteq y$. Therefore we put

$$l = \kappa\left|u'/u''\right| = \kappa y, \qquad \tau_0 = \kappa^2\rho u'^2 y^2$$

and obtain

$$u = (\tau_0/\rho)^{\frac{1}{2}}(\text{const.}+(1/\kappa)\log y). \quad (21)$$

Comparing Eq. (21) with the general Eq. (19), we obtain

$$u = (\tau_0/\rho)^{\frac{1}{2}}[a+(1/\kappa)\log((\tau_0/\rho)^{\frac{1}{2}}y/\nu)]. \quad (22)$$

This equation was given by the present author in 1930. The numerical value found by the author for the universal constant is 0.38. Nikuradse found later that $\kappa = 0.40$ fit the experimental values better. In Fig. 9 are plotted Nikuradse's velocity distribution measurements using $u^+ = u/(\tau_0/\rho)^{\frac{1}{2}}$ and $\log_{10} y^+ = \log_{10}((\tau_0/\rho)^{\frac{1}{2}}y/\nu)$ as coordinates. For $y^+ < 110$ individual points are shown, for higher values of y^+ a certain number of measured points are replaced by their centers of gravity. It is seen that the equation $u^+ = 5.5+5.75\log_{10} y^+$ fits the measurements very well. The factor 5.75 corresponds to $\kappa = 0.40$. There is a systematic deviation for values of

$y^+ = (\tau_0/\rho)^{\frac{1}{2}} y/\nu < 30$. Obviously for small values of the friction distance the similarity assumption leading to the logarithmic law cannot be correct. We remember that at the wall itself the flow is laminar. If we assume a pure laminar sublayer corresponding to the constant value τ_0 of the friction, the corresponding velocity distribution would be given by $u = (\tau_0/\mu) \cdot y$ or

$$u = ((\tau_0/\rho)^{\frac{1}{2}} (\tau_0/\rho)^{\frac{1}{2}} y/\nu), \quad \text{i.e.,} \quad u^+ = y^+. \tag{23}$$

The two lines representing (22) and (23) meet at the point $(\tau_0/\rho)^{\frac{1}{2}} y/\nu \doteq 11.5$. Hence if there were an abrupt transition between pure laminar and turbulent regions, the "thickness of the laminar sublayer" would be given by $y^+ = 11.5$ or $y = \delta_l = 11.5\nu/(\tau_0/\rho)^{\frac{1}{2}}$. Actually the transition is gradual and the value $y^+ = 30$ appears as a reasonable limit for the region influenced by laminar friction.

Before the author found Eq. (22), power laws were used for Eq. (19). If we put

$$u = (\tau_0/\rho)^{\frac{1}{2}} \cdot A \cdot ((\tau_0/\rho)^{\frac{1}{2}} y/\nu)^n, \tag{24}$$

where A is a numerical constant, it is easy to see that a relation must prevail between the exponent n of the velocity distribution and the exponent m in the friction law, mentioned in Section 4. In fact if we solve Eq. (24) for τ_0, we obtain

$$\tau_0 = \tfrac{1}{2}\rho u^2 \cdot \frac{2}{A^{2/(n+1)}} \cdot \frac{1}{(uy/\nu)^{2n/(n+1)}}. \tag{25}$$

Therefore $m = 2n/(n+1)$, or $n = m/(2-m)$. For $m = 1/4$ we obtain $n = 1/7$. We mentioned that for large R.N. m diminishes, as also does n. In Fig. 9 is shown how far the power law formulae with $n = 1/7$ and $1/8$ represent an approximation to the logarithmic formula in certain ranges.

8. Skin Friction in Smooth Pipes

Combining Eqs. (18) and (22), i.e., the developments for the velocity distribution starting from the center and from the wall, the author obtained in 1930 a rather simple formula connecting the frictional loss in pipes and R.N., the *first formula which at the same time is based on theory and checks the experiments.*

Let us consider the velocity u at a point y outside of the laminar range, but near enough to the wall so that the logarithmic formula (22) for the velocity distribution is sufficiently exact. In such a region the two Eqs. (18) and (22) overlap, so that both are valid. Hence we write

$$\frac{u}{(\tau_0/\rho)^{\frac{1}{2}}} = a + \frac{1}{\kappa} \log_e \frac{(\tau_0/\rho)^{\frac{1}{2}} y}{\nu}, \tag{26}$$

and at the same time

$$\frac{U-u}{(\tau_0/\rho)^{\frac{1}{2}}} = f\left(\frac{r-y}{r}\right). \tag{27}$$

If both equations hold, Eq. (27) for small values of y, must have the form:

$$\frac{U-u}{(\tau_0/\rho)^{\frac{1}{2}}} = \text{const.} - \frac{1}{\kappa} \log \frac{y}{r}, \tag{28}$$

and adding (26) and (28), we obtain

$$\frac{U}{(\tau_0/\rho)^{\frac{1}{2}}} = \text{const.} + \frac{1}{\kappa} \log \left(\frac{(\tau_0/\rho)^{\frac{1}{2}} r}{\nu}\right). \tag{29}$$

Let us introduce the friction coefficient referred to the maximum velocity by the relation

$$c_f = \tau_0/\tfrac{1}{2}\rho U^2 \tag{30}$$

and the R.N. referred to the same velocity

$$R = Ur/\nu. \tag{31}$$

Then from Eq. (30) follows immediately

$$(2/c_f)^{\frac{1}{2}} = \text{const.} + (1/\kappa) \log_e (R(c_f)^{\frac{1}{2}}). \tag{32}$$

Eq. (32) can easily be checked by plotting the values of $1/c_f^{\frac{1}{2}}$ as function of $\log (R(c_f)^{\frac{1}{2}})$. If the equation holds, $1/c_f^{\frac{1}{2}}$ appears as a linear function of $\log (R(c_f)^{\frac{1}{2}})$, and the slope of the straight line representing $1/c_f^{\frac{1}{2}}$ as a function of $\log (R(c_f)^{\frac{1}{2}})$ determines the value of the universal constant κ. This plotting is shown in Fig. 10. The circles represent centers of gravity of groups of Nikuradse's measured points. The crosses are representative of mean values taken from measurements of Stanton and Pannell. The straight line corresponds to the equation

$$1/c_f^{\frac{1}{2}} = 3.60 + 4.15 \log_{10} (R(c_f)^{\frac{1}{2}}). \tag{33}$$

The factor 4.15 in the last term corresponds to the value of the constant $\kappa = 0.39$.

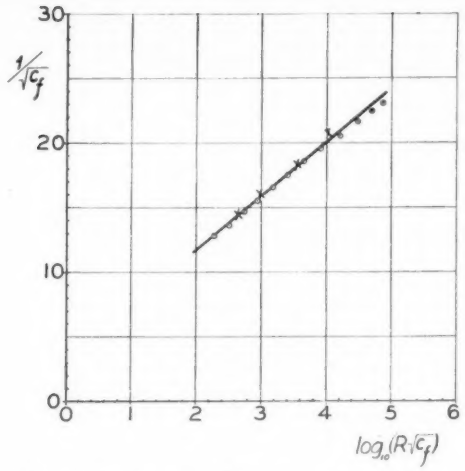

FIG 10. Plotting of $1/c_f^{\frac{1}{2}}$ against $\log_{10}(R(c_f)^{\frac{1}{2}})$ for smooth pipes.

It is easy to show that a formula of the same form with another additive constant is valid for the skin friction coefficient referred to the mean velocity $\overline{U}$ instead of to the maximum velocity U. This remark is due to L. Prandtl.

According to Eq. (27), with the exception of a small region near to the wall, $(U-u)/(\tau_0/\rho)^{\frac{1}{2}}$ is a universal function of y/r, independent of the R.N.; integrating $(U-u)$ over the cross section we obtain

$$\int(U-u)dA=(U-\overline{U})\cdot A$$

so that

$$(U-\overline{U})/(\tau_0/\rho)^{\frac{1}{2}}=\text{const.}$$

or

$$1/c_f^{\frac{1}{2}}-1/\overline{c}_f^{\frac{1}{2}}=\text{const.} \qquad (34)$$

On the other hand the value of $R(c_f)^{\frac{1}{2}}$ is invariable if both R and c_f are referred to the same parameter, i.e., both to the maximum or both to the mean velocity.

The friction coefficient λ used in the hydraulic practice is defined by the relation

$$h=\lambda(l/d)(\overline{U}^2/2g),$$

where h denotes the pressure loss measured in height, l the length, d the diameter of the pipe. The factor λ is the double of the coefficient $\overline{C}_f$. Nikuradse and Prandtl gave, following the author's deductions, the formula

$$1/\lambda^{\frac{1}{2}}=-0.8+2\log_{10}(R\lambda^{\frac{1}{2}}).$$

The factor 2 in this formula corresponds to the value 0.403 for the universal constant κ. The value of κ can be determined from (a) velocity distribution measurements, (b) tests in smooth pipes, (c) tests in rough pipes, (d) friction measurements on plates. Nikuradse gives formulae identical in form to those given by the author previously for the three cases (a)–(c), but he determines the numerical coefficients independently for each case. Comparing his formulae, we find $\kappa=0.40$ for (a), $\kappa=0.403$ for (b) and (c). Plotting his results and those of Stanton and Pannell and Fromm, we find that $\kappa=0.39$ fits both items (b) and (c) well; the same constant is right for the skin friction of plates (d). For the velocity distribution near to the wall $\kappa=0.40$ checks the experiments better, for the velocity distribution in greater distance from the wall $\kappa=0.38$ fits better. In view of the scattering of the measured values and the quite considerable differences between the measurements of different observers, it seems to the author that the accordance between the κ values obtained in different ways is quite satisfactory and substantiates the existence of such a universal constant entering in all four problems mentioned.

9. Skin Friction of Smooth Plates

The results obtained and checked by comparison with pressure drop measurements have been applied by the author to the computation of the skin friction of smooth plates. The basic assumption is that Eq. (32) holds for the turbulent boundary layer, if the R.N. involved in the equation is replaced by the R.N. of the boundary layer, $R_\delta=U\delta/\nu$. The calculation leads to a formula of the form

$$2^{\frac{1}{2}}/c_f^{\frac{1}{2}}=\text{const.}+(1/\kappa)\log(R_\delta c_f) \qquad (35)$$

both for the coefficient of the local friction

$$c_f=\tau_0/\tfrac{1}{2}\rho U^2 \qquad (36)$$

and for the coefficient of the total or mean friction

$$C_f=F/\tfrac{1}{2}\rho U^2 A, \qquad (37)$$

where A is the surface area of the portion considered. The R.N. is referred to the velocity

U outside of the boundary layer and the length x of the portion of the plate considered.

The Eq. (35) settles some questions often discussed without decisive answer in previous technical papers on skin friction. It shows for instance that the contribution of equal areas to the skin friction of a plate diminishes slowly but indefinitely with increasing distance from the leading edge. Also the coefficient of the mean friction diminishes with increasing R.N., i.e., with increasing velocity and increasing length. Formerly it was assumed that both the local and the mean friction coefficients have certain finite inferior limits.

Concerning the numerical values of the constants involved in Eq. (35), the same method of plotting can be used as shown above for the case of the pipes. In Fig. 11 are plotted the

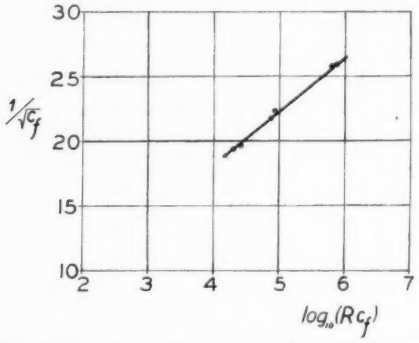

FIG. 11. Plotting of $1/c_f^{\frac{1}{2}}$ against $\log_{10}(Rc_f)$ for smooth plates.

values of the local friction coefficients according to measurements of G. Kempf. These experiments are very remarkable because the values of the local friction were obtained by direct measurements of the force acting on small movable plates arranged at different points of a very long pontoon.

The equation of the straight line representing the measurements is

$$1/C_f^{\frac{1}{2}} = 1.7 + 4.15 \log_{10}(R_x C_f). \tag{38}$$

E. Schoenherr found that the coefficient of the mean friction C_f is given by

$$0.242/C_f^{\frac{1}{2}} = \log_{10}(R_x C_f), \tag{39}$$

corresponding to $\kappa = 0.395$, or

$$1/C_f^{\frac{1}{2}} = 4.15 \log_{10}(R_x C_f).$$

Between the coefficients of the local and the mean friction, Schoenherr found the relation

$$c_f = 0.558 C_f / 0.558 + 2(C_f)^{\frac{1}{2}}, \tag{40}$$

where the numerical constant 0.558 represents $\kappa(2)^{\frac{1}{2}}$.

The calculation leading to Eq. (35) can be found in the author's paper presented at the International Congress for Applied Mechanics (Stockholm 1930) or in his paper on the theory of skin friction in the Proceedings of the Hamburg conference on problems of ship propulsion (1932). The Eq. (35) is obtained by neglecting terms of the order $C_f^{\frac{1}{2}}$ in comparison with $1/C_f^{\frac{1}{2}}$. For moderate values of the R.N. the terms omitted are not quite negligible, but it appears that by slight changes in the constants of the Eq. (35) the whole practical range can be covered with sufficient exactitude by this simpler formula. Prandtl repeated the calculation in a slightly different way and gave the results of his calculation in the *Göttinger Ergebnisse* of 1932. His results are very similar to those obtained by the present author.

Eqs. (38) and (39) have the disadvantage that they cannot be analytically resolved for c_f and C_f. The Table I gives the values of c_f and C_f as function of R.

TABLE I.

R_x	C_f	c_f	R_x	C_f	c_f
2×10^6	0.00614	0.00480	2×10^7	0.00263	0.00222
3	562	444	3	247	216
4	529	419	4	237	210
5	506	402	5	229	196
7	473	379	7	218	187
10^6	441	357	10^8	207	179
1.5×10^6	408	332	2×10^8	189	163
2	387	317	3	179	156
3	360	296	5	167	145
4	342	283	10^9	155	135
5	330	274	2×10^9	141	125
7	311	260	5	127	113
10^7	293	245	10^{10}	118	105

Schlichting suggested that the Eq. (39) can be replaced by an empirical formula $C_f = 0.455/(\log_{10} R_x)^{2.58}$. This formula replaces the Eq. (39) fairly well in the range $10^6 < R_x < 10^9$.

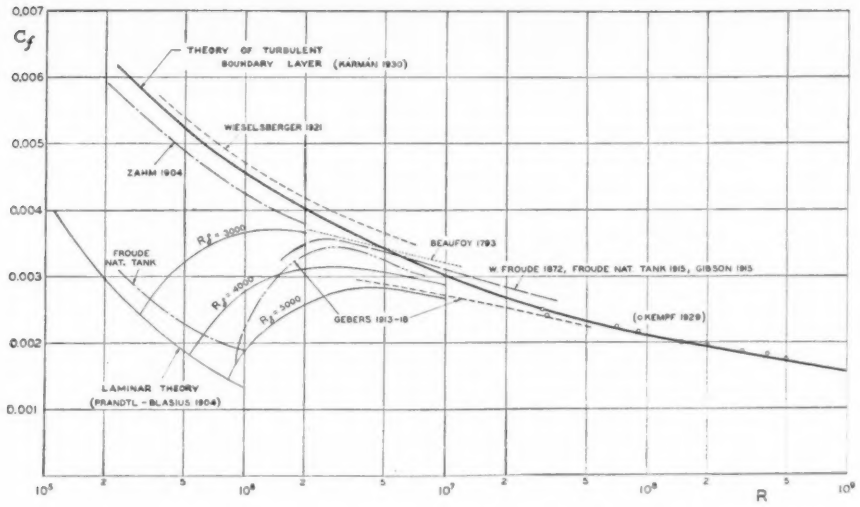

FIG. 12. Skin friction of smooth plates compared with the theory.

10. DISCUSSION OF THE EXPERIMENTS. TRANSITION BETWEEN LAMINAR AND TURBULENT REGION

Fig. 12 represents the results of the most important test on skin friction of smooth plates. The experimental points are, in general, replaced by curves connecting the mean values. In addition, the theoretical curve for the coefficient of the laminar skin friction and the curve for turbulent skin friction corresponding to Eq. (39) are drawn.

The tests of Wieselsberger, Zahm and Gibson were wind tunnel tests on stationary models; all other tests were made by towing flat plates in water. In all tests except those made by G. Kempf the total force acting on the model plate was measured and a correction was made for the additional drag due to the finite thickness of the plate. In Kempf's tests the local friction was measured and the total friction computed by integration.

It is interesting that in the range $2\times10^5 < R < 10^6$ the results of the wind tunnel tests check with the theoretical values for turbulent friction and the tests of the Froude national tank check approximately with the theoretical values for laminar friction. Geber's results in the range $10^6 < R < 10^7$ show transitions between the two theoretical curves.

The transition between laminar and turbulent boundary layers along a flat plate was investigated by Burgers, Hegge Zynen, Hensen, Fage, and recently by Schoenherr. The ideal case of a mathematically thin plate can be approximated by using plates with sharp leading edges. It is to be expected that under similar conditions the transition depends on the R.N. of the boundary layer $R_\delta = U\delta/\nu$. Hegge Zynen and Burgers found that in their wind tunnel experiments the critical value of R_δ was between 1650 and 3500; Hansen found the value $R_{\delta c} = 3100$. It appears that there is no definite characteristic critical value of R_δ for the transition; the value $R_{\delta c}$ depends on the amount of perturbations, for instance on the turbulence in the outside stream. Schoenherr found that the transition was substantially facilitated by producing artificial turbulence in the water upstream from the leading edge. He obtained the transition curves (*b*) and (*c*) shown in Fig. 13 under normal conditions and the curve (*a*) for artificial turbulence produced by roughening the forward edge of the plates. He also found that if artificial turbulence is produced in the water upstream from the leading edge the skin friction values, from $R = 2\times10^5$ on, fall on the curve for pure turbulent friction, as in the tests of Zahm and Wieselsberger. In general, rounding the leading edge has a similar influence.

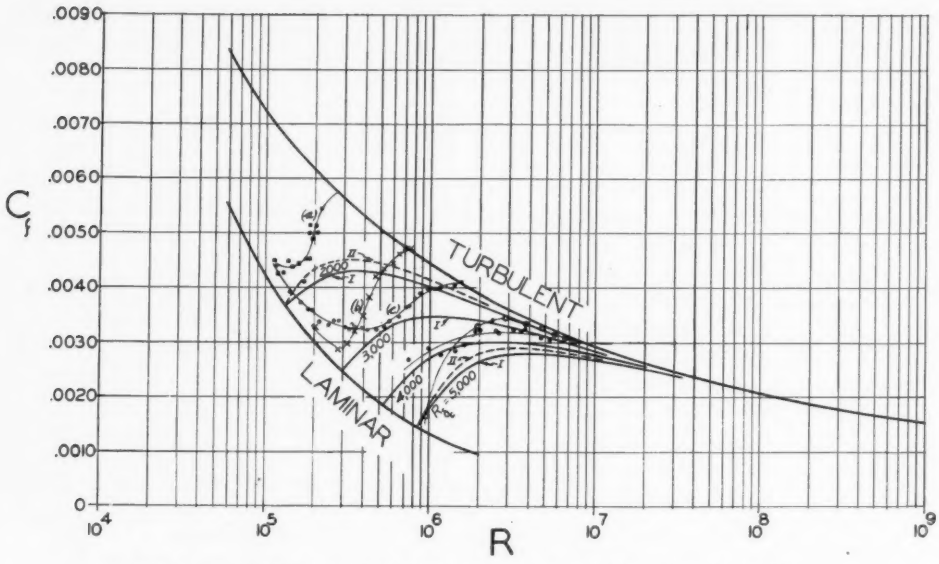

FIG. 13. Transition between laminar and turbulent friction (smooth plates).

The range of critical values of R_δ can be assumed to be between 1600 and 6000.

Assuming that the transition takes place abruptly at a certain value of R_δ a family of "transition curves" for C_f can be calculated as function of the R.M. of the plate with the critical value $R_{\delta c}$ as parameter. Such curves are drawn in Fig. 13. Curves I and II correspond to somewhat different assumptions. In both cases a laminar layer is assumed, until a certain value of R_δ is reached. The turbulent layer beyond the transition point is calculated in case I, as if it had started with the thickness zero at the leading edge; in case II it is assumed that at the transition point the momentum carried by the boundary layer remains unchanged. Geber's experimental curves fit the calculated curves quite well. However, in view of the Schoenherr curves it must be remarked that the transition phenomenon is probably too unstable to be represented by a simple equation.

11. PRACTICAL APPLICATIONS

Predictions of the maximum speed of airplanes based on wind tunnel tests involve a knowledge of the "scale effect" for the drag. Induced drag has practically no scale effect, also the drag of protruding parts follows fairly well the so-called quadratic law, at least beyond their critical R.N., which is connected with a more or less sudden drop of the drag coefficients. Thus the greatest uncertainty is that due to the scale effect on skin friction, i.e., the gradual drop of the friction coefficient with increasing R.N. The range of extrapolation is comparatively large in the case of modern fast airplanes. The comparison can be based on the R.N. of the wing defined in terms of the speed, the mean chord of the wing and the kinematic viscosity. Present atmospheric wind tunnels, with the exception of the Langley Field full scale tunnel, reach values of about $R=2\times10^6$, the variable density tunnel about 3×10^6. The full scale values for large modern transport airplanes may reach 25×10^6, somewhat smaller values corresponding to fast military planes. Hence there is a large change in the skin friction coefficient which influences the predicted speed very noticeably, especially because, in the case of modern clean airplanes, the skin friction may represent the major portion of the total drag.

An exact calculation is made difficult by two facts:

(a) The separation of the skin friction and the drag of protruding parts is hardly possible.

(b) The skin friction acts partly on the wings, partly on the fuselage and tail; these parts have different R.N. and so somewhat different scale effects.

At the California Institute of Technology the following procedure, suggested by Clark B. Millikan, is in use: Two different values for the full scale drag coefficient are calculated, a first value based on the assumption that the total parasite drag follows the skin friction law for smooth plates, and a second value corresponding to the assumption that the profile drag of the wings follows the law mentioned, while for the remainder of the drag the scale effect can be neglected. It is believed that these values constitute limits between which the value of the actual drag should lie.

As an example the following data are quoted, taken from the results of actual wind tunnel and flight tests on an observation type, wire braced, gull wing monoplane.

Model results scaled up without R.N. correction:
$C_{D_p}=0.0396$, calculated $V_{max}=180$ m.p.h.
Wing profile drag only corrected to full scale:
$C_{D_p}=0.0364$, calculated $V_{max}=186$ m.p.h.
Total parasite drag considered as turbulent skin friction and corrected to full scale:

$C_{D_p}=0.0268$, calculated $V_{max}=205$ m.p.h.

Flight tests on the actual airplane gave $V_{max}=196$ m.p.h., i.e., almost exactly halfway between the two extrapolations. The relative distance from the two limits should, in general, be capable of estimation from the general "cleanness" of the airplane investigated. More refined methods of making such estimates have recently been developed by Millikan and will be presented in a paper to be published by him in the near future.

Another important application of the skin friction theories is an approximate prediction of the boundary layer thickness or of the velocity distribution near the surface of plates or streamline bodies. To be sure in the case of streamline bodies only a rough approximation can be obtained, because the "development of the boundary layer" is influenced by pressure distribution and curvature along the surface. However, in many cases, an approximate calculation is of great value, for instance for an estimate of the drag of protruding parts in the boundary layer region, for an estimate of the influence of small roughness, or of the influence of the velocity field near the hull on the behavior of fins, rudders and propellers of airships.

A simple formula for the boundary layer thickness can be obtained by comparing the two expressions for the coefficient of the local skin friction of a plate at the distance x from the leading edge, according to the Eqs. (33) and (38). We have the following two relations:

$$1/C_f^{\frac{1}{2}}=1.7+4.15\log_{10}(R_xC_f) \tag{41}$$

according to (38) and replacing in (33) R by R_δ

$$1/C_f^{\frac{1}{2}}=3.6+4.15\log(R_\delta(C_f)^{\frac{1}{2}}). \tag{42}$$

By subtracting the two expressions it is seen that the value $R_\delta/R_x(C_f)^{\frac{1}{2}}$ is constant and

$$R_\delta=0.38R_x(C_f)^{\frac{1}{2}} \quad \text{or} \quad \delta=0.38x(C_f)^{\frac{1}{2}}. \tag{43}$$

The value of C_f as function of R_x is tabulated on page 12; hence the value of δ can easily be computed. The velocity distribution in the boundary layer is approximately given by the formula

$$u=U[1-4.15(C_f)^{\frac{1}{2}}\log_{10}(\delta/y)]. \tag{44}$$

Let us calculate, as an example, the boundary layer thickness at the trailing edge of an airfoil; with chord of 3 m = 9.8 ft. for the speed $U=100$ m/sec. = 223.7 m.p.h. The corresponding R.N. (for "standard" air) = $20{,}000{,}000=2\times10^7$; $C_f=0.00222$, accordingly $\delta=5.4$ cm.

12. Skin Friction of Bodies of Revolution (Airships)

The application of the boundary layer theory to bodies of revolution is interesting for analyses and predictions of airship drags. The resistance of a bare airship hull is almost entirely due to skin friction, the pressures at the bow and the stern being practically balanced. Early wind tunnel tests, carried out at comparatively low R.N., appeared to be inconsistent with one another because the transition between laminar and turbulent flow was not taken into consideration. The airstreams in different wind tunnels

have very different degrees of turbulence and so the transition occurs in quite different ways in different wind tunnels.

B. M. Jones and H. L. Dryden independently showed that such differences in airstream turbulence could account qualitatively for the wind tunnel discrepancies. In their considerations a figure of revolution was replaced by an "equivalent flat plate" and the existing information on the flow along flat plates was thus applied to their problem. Shortly afterwards C. B. Millikan applied the boundary layer integral relation (analogous to Eq. (2), but taking the variation of pressure along the surface into account) to the case of axial flow about a figure of revolution and gave expressions for the variation of boundary layer thickness and skin friction along such a body, assuming a parabolic velocity profile for the laminar portion of the boundary layer and a "1/7th power" profile for the turbulent portion. The calculations were carried out in detail for the so-called N.P.L. models and curves of drag coefficient *vs.* R.N. for various degrees of airstream turbulence were obtained. These gave a satisfactory quantitative agreement with many of the experimental curves, so that the explanation of the latter's inconsistencies was in this way successfully accomplished.

Miss H. M. Lyon has carried the experimental side of the problem still further in an elaborate investigation of the correlation between the drag coefficient *vs.* R.N. curves for two airship hull models and the degree of turbulence produced artificially in the airstream.

Freeman repeated Millikan's calculations for the hull of the Akron dirigible and obtained a remarkably close agreement with his beautiful measurements on a 1/40th scale wind tunnel model, both as to drag coefficient and distribution of boundary layer thickness. For the R.N. of his measurements the 1/7th power law used in Millikan's theory was shown to be a very satisfactory approximation to reality. However, for the extrapolation of drag values to full scale and for full scale boundary layer calculations the 1/7th power law will almost certainly be a much less satisfactory approximation, so that it would be very desirable to repeat the analysis by using our improved knowledge of the laws of skin friction.

13. The Influence of Roughness

The problem of the influence of surface roughness on the skin friction is rather complex. Although many experiments have been conducted concerning the friction in rough-walled pipes, considerably fewer tests are available on skin friction of rough plates in a free stream. In pipes with different diameters but with identical or geometrically similar surface conditions the coefficient of skin friction depends on the R.N. and the "relative roughness." The relative roughness is the ratio between a length which is characteristic of the roughness and a length characteristic of the cross section (for instance, the hydraulic radius). It is not possible to define *a priori* which dimension is to be considered characteristic of the roughness, except that it must be the same for identical surfaces and for walls with geometrically similar kinds of roughness proportional to the scale of the protruding parts (for instance, to their mean height). Denoting this parameter by k and the radius by r, the friction coefficient will be

$$c_f = F(R, k/r). \tag{45}$$

The general form of this function is not known, in fact it is not even sure that one universal function covers the case of all kinds of rough walls. It is more probable that it depends on the shape of the roughness. Fortunately, the experimental evidence shows that the conditions are relatively simple for high R.N. and comparatively large relative roughness. It appears that with increasing R.N. the skin friction coefficient becomes independent of the viscosity and in turn independent of the R.N. In this respect the behavior of rough surfaces is fundamentally different from that of a smooth surface, since in the case of the smooth surface the friction coefficient decreases indefinitely with increasing R.N.

For the range in which the friction coefficient is independent of the R.N., a simple relation between the friction coefficient and the relative roughness can be found, based on the velocity distribution laws given in Section 6. We remember that the ratio (velocity defect/friction velocity) was found to be a universal function of the ratio (distance/radius), and it has been

noted that this theorem holds also in the case of rough surfaces. Accordingly, denoting again the maximum velocity by U, the velocity at the distance y from the wall by u, and the wall friction by τ_0, we write

$$(U-u)/(\tau_0/\rho)^{\frac{1}{2}}=f(y/r). \tag{46}$$

The development for the velocity distribution starting from the wall will be different in the cases of smooth and rough walls. For smooth walls we assume that u is a function of y, τ_0, ρ and μ. In the general case of a rough surface u will be a function of these quantities and also of the roughness parameter k. However, the fact that for large R.N. the skin friction coefficient is independent of the viscosity indicates that the velocity distribution is also independent of ν and hence u depends only on y, τ_0, ρ and k. Dimensional analysis shows that the only form possible for this functional relation is

$$u=(\tau_0/\rho)^{\frac{1}{2}}g(y/k). \tag{47}$$

On the other hand, we obtained from the similarity of the turbulent flow pattern the relation

$$u=(\tau_0/\rho)^{\frac{1}{2}}(\text{const.}+(1/\kappa)\log y) \tag{48}$$

so that due to (43) the development for the velocity distribution starting from the wall will be

$$u=(\tau_0/\rho)^{\frac{1}{2}}(\text{const.}+(1/\kappa)\log (y/k)). \tag{49}$$

Combining Eqs. (46) and (49) we obtain, by a reassumption similar to that applied in Section 8

$$U=(\tau_0/\rho)^{\frac{1}{2}}(\text{const.}+(1/\kappa)\log (r/k)) \tag{50}$$

and

$$(2/C_f)^{\frac{1}{2}}=a+(1/\kappa)\log (r/k). \tag{51}$$

It is easy to show (compare Section 8), that the same equation holds for the coefficient referred to the mean velocity.

The Eq. (51) was found by the author in 1930, by a somewhat different reasoning from that given above. The equation can be checked independently of the definition of k by tests in pipes or channels with walls of identical surface conditions.

Let us write Eq. (51) in the form

$$2^{\frac{1}{2}}/c_f^{\frac{1}{2}}=a-(1/\kappa)\log k+(1/\kappa)\log r. \tag{52}$$

Then obviously, if we plot $1/c_f^{\frac{1}{2}}$ against $\log r$, we get parallel straight lines, each line corresponding to a given roughness. The slope of these lines is determined by the universal constant κ, which was encountered also in the case of smooth pipes and smooth plates.

In Fig. 14 are plotted results of tests made by Fromm and Nikuradse. With one exception they agree with the foregoing conclusions. The exceptional case refers to a test in which a surface with extreme roughness (sawteeth of 1.5 mm height) was used.

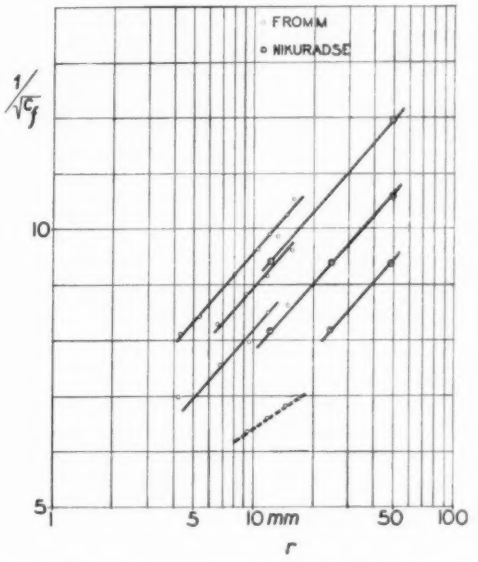

FIG. 14. Plotting of $1/C_f^{\frac{1}{2}}$ for channels of different widths.

The vertical distance between any two such straight lines is determined by the difference of the terms $(1/\kappa)\log k$, i.e., by the ratio of the roughnesses. Thus, by choosing one definite roughness as a basis for comparison, it is possible to assign a definite value to any surface. Nikuradse produced geometrically similar rough surfaces for all of his tests, making use of sand grains of various diameters. He found that by determining the roughness by Eq. (52), the roughnesses of the surfaces used by him are simply proportional to the diameter of the grains used. Thus we can tentatively use as a unit of roughness the grain diameter of an equivalent

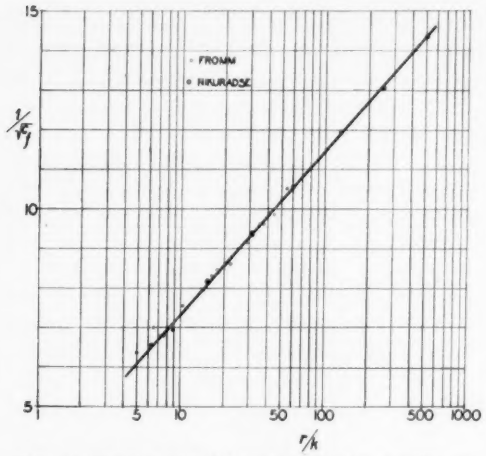

FIG. 15. Plotting of $1/C_f^{\frac{1}{2}}$ against relative roughness.

granular surface. Fig. 15 shows the results of the Fromm and Nikuradse tests combined into one straight line, which has the equation

$$1/c_f^{\frac{1}{2}} = 3.0 + 4.15 \log (r/k). \qquad (53)$$

The next step is the application of Eq. (51) to the problem of skin friction of rough plates, in the same manner as we applied Eq. (33), deduced for the skin friction in smooth tubes, to the case of smooth plates. We must replace the radius r in Eq. (51) by the boundary layer thickness. First, we notice that the "relative roughness" k/δ which determines the friction is variable for constant k, it is large for small values of δ (i.e., near the leading edge), and decreases with increasing distance from the leading edge. Hence the validity of such a calculation is restricted to cases in which the roughness k is so large that the assumption of a friction independent of the R.N. holds.

We obtain, under these conditions, the equation for the local friction coefficient:

$$1/c_f^{\frac{1}{2}} = 5.8 + 4.15 \log ((x/k)c_f^{\frac{1}{2}}). \qquad (54)$$

As might be expected, the friction depends on the ratio k/x (roughness relative to the length). In the following table are calculated a few corresponding values of c_f and x/k:

$c_f =$	0.0040	0.0035	0.0030	0.0025	0.0020
$x/k =$	4×10^3	8×10^3	18×10^3	53×10^3	220×10^3

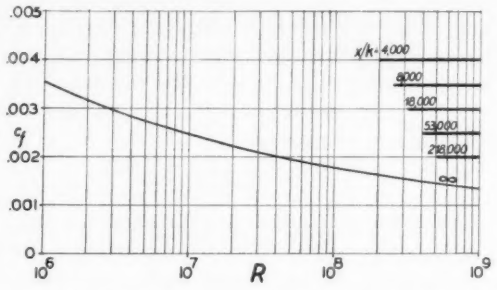

FIG. 16. Local friction coefficients for rough plates.

These values give a fair estimate of the influence of roughness on the coefficient of skin friction for plates. Fig. 16 shows these values of c_f for various roughnesses, compared with the curve for smooth plates. For a more exact calculation, it must be taken into account that in general the local skin friction will be a function both of the boundary layer R.N. and of the relative roughness. Experimental evidence for the range in which both the R.N. and the roughness have an influence on the friction is somewhat contradictory, even in the case of pipes. However, the interpretation of the experimental results is relatively easy in the case of the artificial granular roughness used by Nikuradse. He found:

(a) that the influence of roughness is, for all practical purposes, null below a certain R.N., and that the limiting value of the R.N. increases with decreasing k.

(b) that beyond this limit the friction coefficient, for a given roughness, as a function of the R.N., first decreases, then slowly increases and becomes constant at a certain value of the R.N., which value also depends on the roughness.

Considering the two limits (the limit at which the influence of the roughness appears, and the limit beyond which the friction coefficient is constant) Nikuradse's tests show that both correspond to definite values of $(\tau_0/\rho)^{\frac{1}{2}}k/\nu$. This can easily be understood: Let us remember that in the case of a smooth wall, we have in the immediate neighborhood of the wall a laminar region, whose outer limit or "the thickness of the laminar layer" δ_l is proportional to $\nu/(\tau_0/\rho)^{\frac{1}{2}}$; thus the parameter $(\tau_0/\rho)^{\frac{1}{2}}k/\nu$ is proportional to k/δ_l or to the ratio between the roughness and the thickness of the laminar sublayer.

According to the tests, the influence of the roughness starts with $(\tau_0/\rho)^{\frac{1}{2}}k/\nu \cong 3$, and the friction coefficient becomes constant for $(\tau_0/\rho)^{\frac{1}{2}}k/\nu > 60$. We remember that $\delta_l \cong 11.5\nu/(\tau_0/\rho)^{\frac{1}{2}}$ gives about the "thickness" of the laminar sublayer. Thus it appears that the influence of roughness becomes noticeable if the height of the roughness has the same order of magnitude as the laminar sublayer ($k/\delta_l \gtrsim \frac{1}{4}$) and that the viscosity has no more influence if the roughness is large in comparison with this thickness ($\kappa/\delta_l \gtrsim 6$).

Accepting this evidence for the influence of roughness with the limiting value $(\tau_0/\rho)^{\frac{1}{2}}k/\nu = 3$, we can carry out a quite interesting calculation indicating the magnitude of "allowable roughness" in practical cases—"allowable" in the sense that roughness below this limit does not influence the friction of a plate.

According to this condition the limiting value of k is given by

$$k = 3\nu/(\tau_0/\rho)^{\frac{1}{2}} = 3(\nu/U)(2/c_f)^{\frac{1}{2}}. \qquad (55)$$

Now, for smooth plates c_f is a function of the R_x and is calculated on page 12. Using these tabulated values, we obtain for the case of an airfoil with 3 m chord and 100 m/sec. speed (x=distance from the leading edge), Table II.

TABLE II.

x	k	x	k
0.075 m	0.0100 mm	1.5 m	0.0129 mm
0.1	0 0103	2.0	0.0131
0.25	0.0111	2.5	0.0133
0.5	0.0118	3.0	0.0135
1.0	0.0125		

Similarly, for an airship of 200 m length and speed of 40 m/sec., we have Table III.

It is interesting to notice that recent experiments of the N.A.C.A. give the same order of magnitude for the allowable roughness of an airfoil.

It is evident that a certain small roughness is more harmful near the leading edge and less harmful at the rear portion of the surface. This has been proved experimentally by G. Kempf and corresponds to common experience.

TABLE III.

x	k	x	k
0.075 m	0.0230 mm	2 m	0.0306 mm
0.1	0.0237	2.5	0.0311
0.25	0.0257	3	0.0316
0.5	0.0276	10	0.0344
1.0	0.0290	50	0.0385
1.5	0.0299	100	0.0400
		200	0.0418

Another consequence which can be drawn from Eq. (55) is the fact that the allowable roughness k depends chiefly on the speed and only to a smaller degree on the R.N. This enhances the reliability of wind tunnel tests, since it shows that in model scale-reduction it is not necessary to reduce k in the ratio of the length scale, but only in the ratio in which $c_f^{\frac{1}{2}}$ is increased.

Unfortunately the evidence of tests on pipes with different roughnesses produced by various paints or by different manufacturing methods is not so consistent as are Nikuradse's tests on artificially roughened pipes. Some of these other tests are very exact, hence it is probable that Nikuradse's conclusion as to the existence of a universal "roughness function" does not cover all kinds of roughness. It is probably that the one parameter k is not sufficient in itself to characterize the surface; at least one more parameter, such as the mean distance between protruding parts, must enter into the picture. At the present time it is not possible to compute the skin friction for a plate with a given roughness.

The author believes that the main problems of future research in the field of skin friction are:

(a) to work out a method of determining a reliable "roughness scale" for different kinds of materials used in practice,

(b) to investigate, by systematic experiments on rough surfaces, the transition range, i.e., the range in which both the R.N. and the relative roughness influence the friction.

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Air Force Measurements on Bodies Moving Through Still Air

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(Received October 30, 1933)

THE six aerodynamic reactions, consisting, in the most general case, of three forces and three moments which are sustained by a body moving through a fluid or by a body held in a fluid stream, may be classified on the basis of their origin into two general groups, those which are due essentially to surface pressure and those arising principally from surface friction. Wing lift and pitching moment are common reactions of the first group, while profile drag and yawing moments are examples of the second. Generally speaking, the pressural reactions on a body of easy form and low incidence are very nearly the reactions associated with potential flow and hence are quite closely proportional to the square of the velocity. They are, therefore, essentially free of scale effects and possess a certain degree of stability against the various disturbances which are found in the air streams of wind tunnels. In such cases, the pressural reactions which are measured on a model in a wind tunnel, when multiplied by the square of the ratios of scale and speed, are the reactions on the full scale body. There are other cases, however, where the pressural reactions as measured in the wind tunnel have no simple relationship to the full scale values.

When the burble attitude[1] or the critical bluntness[2] of a body at which the flow breaks away from the surface is approached, the pressural reactions become unstable and the flow regime becomes sensitive to various disturbances such as the turbulence initially present in a tunnel stream.[3] In these critical cases the close measurement of the pressural reactions on a body in a wind tunnel, regardless of its size, are either of doubtful value or are distinctly deceptive and cannot be safely used as a basis for the study of important dependent phenomena such as spinning or lateral control at the stall.

The frictional reactions present a more serious problem because of their sensitivity to turbulence and other tunnel disturbances not only at the burble attitudes and critical fineness ratios, as in the case of the pressural reactions, but at all other attitudes and fineness ratios as well. The general dependence of the frictional reactions at a given Reynolds number upon tunnel disturbances is due essentially to the effect of such disturbances upon the position of the transition region which separates the laminar and turbulent phases in the boundary layer. The most important imperfections of the wind-tunnel air stream in this connection are the presence of pressure gradients alongstream and initial stream turbulence. The effect of the first is to retard the transition and that of the second to advance it, the latter effect normally being much the more important. Other disturbances, all of which tend to advance the transition, although to a lesser extent and hence to increase the drag coefficient at a given Reynolds number, are shrill or explosive sounds, fluttering of the body or of the air stream pressure and high frequency vibrations of the body surface. Fortunately, these latter disturbances can all be practically eliminated without difficulty from wind-tunnel air streams.

The uncertainty in the position of the transition from the laminar to the turbulent boundary layer phase, as it is effected by these disturbing factors, is quite different for different bodies and at different Reynolds numbers. The transition position for flat plates, expressed in terms of

[1] A wind burbles, for example, at $-6°$ or $-8°$ angle of attack referred to the null line and again at $+14°$ or $+16°$.

[2] The critical bluntness of a spindle form or airship hull is somewhat uncertain but appears to be near a fineness ratio of two.

[3] For the effect of tunnel turbulence on wing burbling, for example, see *Effect of Turbulence*, by Millikan and Kline, Aircraft Engineering, August (1933).

the transition Reynolds number,[4] may occur, for example, at any value between 10^5 and 5×10^5 depending upon the kind, the amount and the number of the disturbing factors.[5] A plate nine inches wide in an air stream whose velocity is 100 feet per second may have a boundary layer, therefore, which is all laminar or 20 percent laminar and 80 percent turbulent, depending upon the wind tunnel and the model mounting, the relative drag coefficients being for the two cases about one to two.

In the case of spindle forms, such as airships, which have a surface point of minimum pressure near the bow, followed by a region of rising pressures and usually by a second point of minimum pressure near the stern, the position of the transition region is not only very sensitive to disturbing factors, over the common wind-tunnel range of Reynolds numbers but also tends to be unstable. The instability at these Reynolds numbers is due to a tendency of the transition region to leap upstream across the midship area of rising pressures to the forward minimum pressure point when turbulence appears or increases in the air stream. Such a shift in the transition region may double the resistance coefficient of the body at a given Reynolds number. Under such circumstances the accurate forecasting of full scale resistance coefficients for airships from measurements made in wind tunnels is quite hopeless.

For these reasons the initial turbulence in tunnel air streams has come to be recognized by present day experimental aerodynamics as an intractable if not an insolvable defect in the wind-tunnel method. Notwithstanding notable advances which have been made, turbulence still cannot be successfully measured or prevented, neither can it be completely controlled or defined. Its existence in varying degrees in different tunnels continues to be the chief cause of a variety of inter-tunnel disagreements and inconsistencies.

In order to evade the whole turbulence problem, the Massachusetts Institute of Technology became interested early in 1930 in conducting tests in calm, non-turbulent air. Under the sponsorship of the late President Stratton, who was chiefly interested in the project, plans were laid for the use of the proposed model basin, the completion of which was then in prospect. It was considered advisable, however, to conduct some preliminary studies in the interim. After several methods were discussed the following apparatus was agreed upon for these preliminary investigations.

There exists at the Institute a large room, or rather open well, in the Steam Engineering Laboratory, the size and shape of which may be seen in Fig. 1. Although the air here is more or less disturbed during the day due to convection currents from hot steam engines on the lower levels, drafts from open windows, etc., the air is very quiet and the temperature quite steady late at night after the building has been shut up for some hours. A number of observations of smoke from titanium tetrachloride have shown that from about 11 P.M. until 6 A.M. the maximum velocity of the air in any direction is about 0.2 of a foot per second and that it is often so still that no movement can be detected. This motion is regarded as negligible in comparison with the speed at which the tests are run. An attempt to secure a quantitative measurement of the turbulence existing in this body of air with the methods at present available did not seem likely to produce results of much value. Although it is probable that some very slight degree of large scale turbulence is present,[6] the air appears to be as near to the condition of complete rest as it is practical to come with so large a mass of air.

The method agreed upon for the preliminary tests was to move the models through the still air of this large room in a straight line with as nearly a uniform velocity as possible and to measure the speed, the model lift and the drag simultaneously. The models were to be

[4] The transition Reynolds number is defined as VL_t/ν where L_t is the distance from the bow, or the leading edge, to the transition region between the two boundary layer phases.

[5] Some important specific evaluations of the transition Reynolds number are 1.6×10^5 by Elias, N.A.C.A. Tech. Memo. No. 614; 3.2×10^5 by Hansen, N.A.C.A. Tech. Memo. No. 585; and 4.8×10^5 by Burgers and van der Hegge Zignen, Delft (1924).

[6] Tetrachloride smoke, for example, moves in a very leisurely manner and seems to have very little tendency to form eddies or whorls.

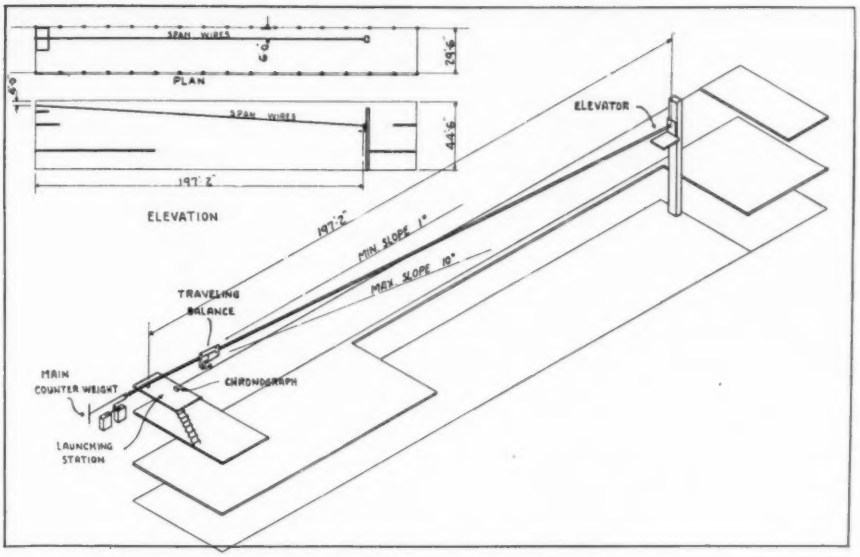

FIG. 1. The site of the coasting apparatus with principal dimensions.

mounted on a suitable carriage running on two long and very taut inclined wires; the model and carriage were to be launched at nearly the terminal velocity, allowed to coast down the wires and finally brought to rest by a suitable retarding gear. The absolute speed was to be measured with an electric tape chronograph, while the lift and drag forces were to be recorded as continuous records on a smoked glass chart borne and rotated by the carriage, and the two records correlated. The drag mechanism, at least, was to be in dynamic balance and quite independent of the lift mechanism. Both were to be oriented accurately to each other and to the wire tracks.

The final apparatus which was built to these general specifications is shown in Fig. 1 which outlines the wire site, and in Fig. 2 which gives a close view of the travelling balance. The main span is of two 1/8-inch high strength alloy steel wires, one placed 8 inches above the other and in horizontal projection is 213 feet long. The wires are wedged into steel blocks at the ends in such a way that the fastening will develop the full strength of the wire. At the lower end of the span the wire anchor blocks are fastened to an elevator which is supported on a structural steel tower. This elevator is suitably guided on the tower and is provided with a motor, counterweight, etc., so that it may be driven up or down the tower under its own power by an operator who rides with it. By moving the elevator up or down the slope of the span wires may be changed from approximately 1° minimum to 10° maximum. Wire tautness, sufficient to reduce the bare wire sag to about one inch, is maintained by two 850-lb. double reeved weights, which are attached to the upper end of the span wires by means of a suitable sheave arrangement.

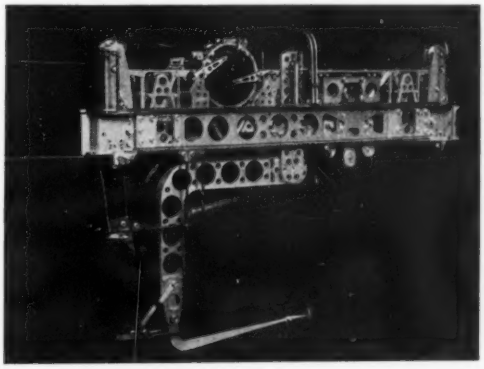

FIG. 2. The travelling balance ready to run.

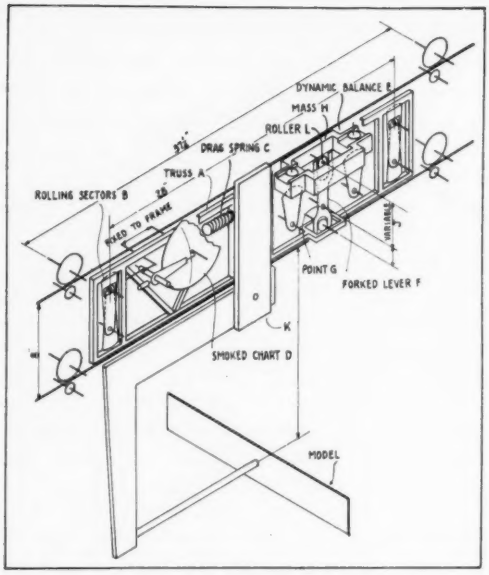

FIG. 3. The drag system of the travelling balance.

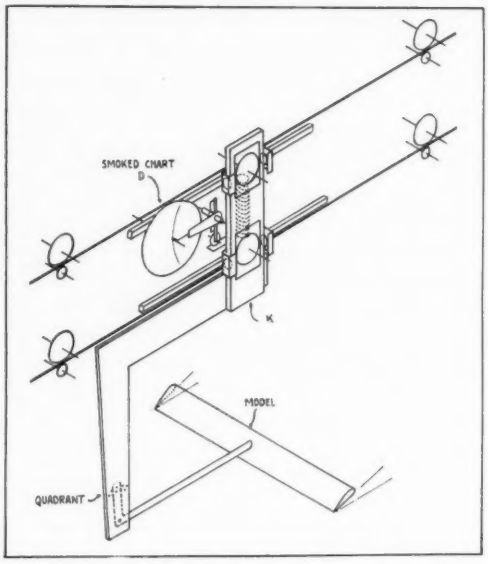

FIG. 4. The lift system of the travelling balance.

The travelling balance, Fig. 2 and shown diagrammatically in Figs. 3 and 4, performs the following functions: It carries the model; it resolves the aerodynamic reaction on the model accurately into its lift and drag components and records their values; it sends out signals at exactly equal distance intervals along the span and simultaneously makes a tally mark on the smoked glass disk by which the records of speed and air forces can be correlated. The drag is measured by the displacement of the drag truss *A*, Fig. 3, which is supported on four rolling sectors *B* against the drag spring *C* and is recorded on the smoked chart *D* through a fine wire and a pointer. A vital part of the drag mechanism is the dynamic balance, marked *E* in the figure, whose function is to suppress all effects of accelerations in the direction of flight. It consists essentially of a mass supported on two rolling sectors and closely guided so that its path and that of the drag truss are accurately parallel. This mass carries a roller which works in one end of a forked lever, *F*; the other end of the forked lever works on a second roller which is fastened to the drag truss. By this arrangement the acceleration force of the total mass of the drag truss system acting at the truss roller and the corresponding force of the mass *H* acting at roller *L* can be made to give equal and opposite moments about the point *G* by a simple adjustment of the arm length *GL*. When roller *L* is so adjusted by means of the micrometer screw provided for the purpose, the drag system is free of all effects of translational accelerations in the direction of the drag. No attempt was made to balance the moments due to angular accelerations, since such accelerations are negligibly small.

The lift is measured by the displacement of the lift system, Fig. 4, against the lift spring. Member *K* is carried by the drag truss but is free to move perpendicular to it, the displacement being recorded by a pointer system on the same smoked chart that records the drag truss displacement.

Particular attention has been given to the reduction of the pitching of the travelling balance to a value that will not impair the precision of the drag measurements through an error in the force resolution. For this reason the carriage has been made quite long and is designed to carry its models below the center section so that the resultant forces on lifting models will pass near the center of gravity of

the carriage and induce a minimum of pitching.

The entire instrument, Fig. 2, is constructed from rolled duralumin shapes and high strength aluminum alloy castings. It is very light, weighing only 14.5 pounds with housing, yet is sufficiently rugged and stiff to maintain its internal alignments and adjustments in normal usage and, in the case of an emergency, to withstand without serious injury a force of 200 or 250 pounds, corresponding to a deceleration of 15 or 18g, applied at the bow. All working parts are fully shielded from the air by a transparent celluloid housing streamlined and proportioned to render the interference effects on the model negligibly small.

The practical speed range of the travelling balance is from 5 to 40 feet per second and is controlled by adjusting both the launching speed and the wire slope and, when necessary, by adding aerodynamic drag to the carriage. No difficulty is found in adjusting the speed so that it is uniform within ± 0.1 foot per second over the central span region. The small wire sag gives a slight decline in the gravity towing component along the span which introduces a small but very useful variation in the velocity, since it makes it possible by launching slightly under the terminal speed to obtain a speed maximum, and hence speeds of nearly constant magnitude, in the mid-span region.

The method of measuring speeds is simple and accurate. As the travelling balance advances, a trolley attached to the balance frame and held in contact with the upper wire by spring pressure, rotates the glass chart, through a suitable gear train, at the rate of one revolution per run. It also rotates a Bakelite cam with a metal insert at the rate of twenty revolutions per run. When the cam follower makes contact with the metal insert, the chronograph circuit is closed and a mark is made on the chronograph tape. Simultaneously, a scriber on the cam follower shaft makes a sharp break in its record line on the smoked chart, Fig. 5. Since the tape speed is constant—three, six or twelve inches per second depending upon the need—the length of tape between any two adjacent marks is a measure of the average velocity for the corresponding interval of span. The part of the smoked glass record which was made in passing the particular

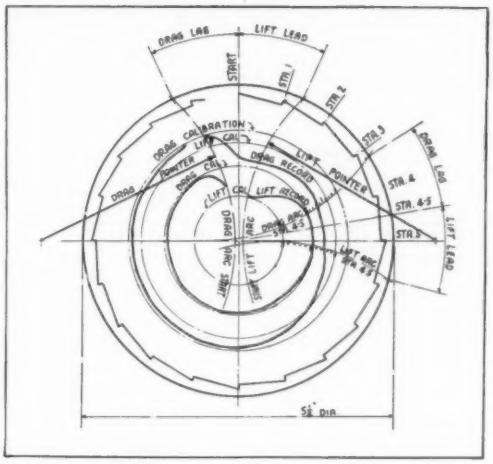

FIG. 5. A typical smoked glass record.

interval can be easily identified. Under normal conditions when the speed is nearly constant, the speed precision is approximately ± 0.5 percent. The chronograph circuit[7] is shown in Fig. 6.

The gear for launching the travelling balance at various speeds is shown diagrammatically in Fig. 7. The energy for launching is supplied by a falling 500-lb. weight which transmits its energy to the balance through a system of pulleys and cables, as shown in the figure. The launching acceleration is about 4g which yields a maximum speed of about 40 feet per second when the catapult travels its maximum distance. For intermediate launching speeds the catapult is pulled back to an intermediate position measured on a suitable fixed scale.

The arresting mechanism, shown diagrammatically in Fig. 8, is divided into two parts,

[7] The chronograph circuit arrangement is as follows: The two insulated span wires are connected into the grid circuit of a vacuum tube amplifier so that the "make" puts a heavy negative bias on the grid, thus allowing current to flow in the plate circuit and operate the chronograph pen. The arm which carries this pen is provided with an interrupter which breaks the circuit as soon as the pen moves. The plate circuit is then dead and, since the travelling balance no longer holds the grid circuit closed after passing over the station, the grid takes a positive potential from battery B, Fig. 6, and further operation of the plate circuit is prevented until the next station is reached. In this way the pen is prevented from making more than one jab for each station.

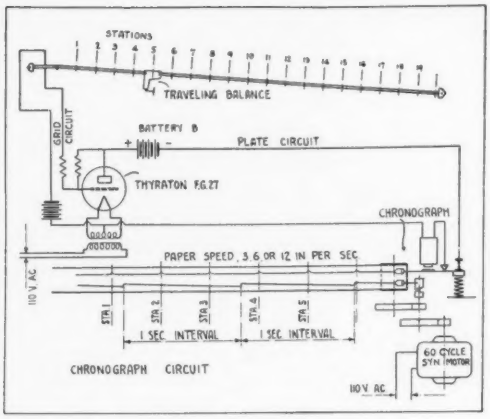

FIG. 6. The chronograph circuit.

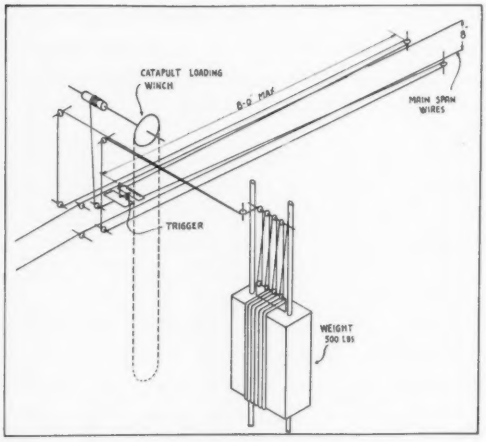

FIG. 7. Diagram of launching gear.

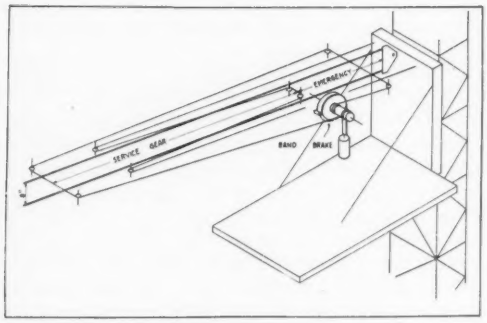

FIG. 8. Diagram of arresting gear.

the service gear and the emergency gear. The service gear consists of two lengths of sash cord which cross the flight path and, after running through a series of pulleys finally wind on a small drum which is provided with a band brake. The band is anchored at one end and pulled tight by a hanging weight at the other. The anchorage is adjustable so that the angle of wrap may be varied by the operator who can thus control the "hardness" of the gear. The emergency gear is behind the service gear and consists simply of three airplane shock cords, as shown.

A typical smoked glass record, with certain information added to indicate the method by which the forces are evaluated, is shown in Fig. 5. The records are regularly read with the aid of a low power microscope mounted on a swinging arm and so located with respect to the record that the path described by the cross hairs coincides with the arcs made by the original lift or drag scribing point. A suitable scale is provided by which the position of a record line which runs between two calibration circles can be quickly and accurately read.

The precision of the drag readings under the conditions of static calibration is ± 1 gram for drags under 50 grams, and ± 2 grams for drags between 50 and 500 grams. The lift readings are reliable under the same conditions to ± 2 grams for loads under 100 grams, ± 3 grams for loads between 100 and 500 grams and ± 5 grams for loads between 500 and 1500 grams. Calibration test runs were made to determine the minimum distance between model and carriage necessary to avoid measurable interference.

As a part of the calibration of the apparatus investigations were carried through on two sets of models, a series of circular disks of varying diameters and a series of equiarea rectangles of varying aspect ratios. All were held normal to the wind. Since the drag of such bodies is entirely pressural in origin and since the point of breakaway is fixed at the body edges with great stability, the drag must be nearly independent of initial turbulence in the air. The wind tunnel and the calm air drags of such bodies should, therefore, agree. It will be seen that the experimental results show an agreement within fair precision.

The series of circular disks, consisting of five disks ranging in diameter from 2 to 13 inches, was made from 1/16″ thick 17st aluminum alloy machined circular to the chosen diameter to a tolerance of ±0.005″. The edges were made square with the thickness of the outer portion reduced so that all disks were in effect geometrically similar with $t/d = 208$. For convenience, the diameters were chosen so that the intervals between the logarithms of the diameters would be equal. This series was tested in the five-foot M. I. T. wind tunnel and on the coaster. The results are shown in Fig. 9. Agree-

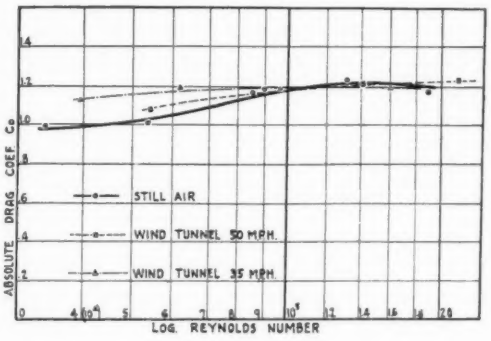

FIG. 9. Drag of circular disks in wind tunnel and in still air.

ment is excellent over a considerable range, well within ±3 percent for Reynolds numbers between 80,000 and 225,000, although at very low Reynolds numbers, the precision of both wind tunnel and coaster begins to decline, due to the smallness of the forces. In Fig. 10 the results of other studies are also given for comparison.

The second series consisted of nine rectangles all having the area 51.8 square inches but varying in aspect ratio from 1 to 40. The calm air drag coefficients are plotted in Fig. 11 along with the coefficients which were measured in a tunnel

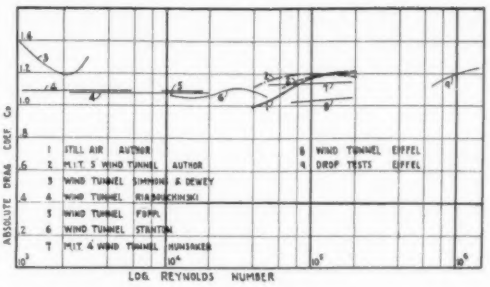
ABSOLUTE DRAG COEF Co
1 STILL AIR AUTHOR
2 M.I.T. 5 WIND TUNNEL AUTHOR
3 WIND TUNNEL SIMMONS & DEWEY
4 WIND TUNNEL RIABOUCHINSKI
5 WIND TUNNEL FAPPL
6 WIND TUNNEL STANTON
7 M.I.T. 4 WIND TUNNEL HUNSAKER
8 WIND TUNNEL EIFFEL
9 DROP TESTS EIFFEL
LOG. REYNOLDS NUMBER

FIG. 10. Drag of circular disks. Comparison of still air results with values reported by others.

by Prandtl[8] and those found from drop tests by Eiffel.[9] The agreement is fair, varying from complete agreement at the aspect ratio 4 to a departure of about 5 percent at the aspect ratio 15 and of nearly 10 percent at the ratio 1, the wind-tunnel values being low. It is noteworthy that the drag coefficients appear to approach the

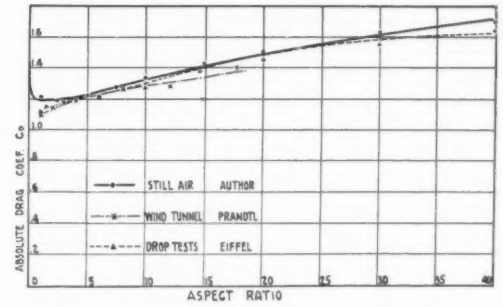

FIG. 11. Drag of rectangular plates. Comparison of coaster results with those reported by others.

value 2.0 asymptotically with the increase of aspect ratios toward infinite values, and that the gradient of the coefficient with aspect ratio at the ratio unity is now found to be zero as it should be.

[8] L. Prandtl, *Ergebnisse der Aerodynamischen Versuchsanstalt zu Göttingen.*

[9] A. Eiffel, *The Resistance of the Air and Aviation.* Translated by J. C. Hunsaker.

Aerodynamics of the Wing Slot

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THE large maximum lift coefficient of a wing equipped with a slot parallel to its leading edge is a discovery of sufficient importance to warrant more study and research than it has hitherto received. The addition of the slot gives the wing a larger lift than one would expect, and further analysis may lead to more efficient slot arrangements. A rational explanation of what happens will not only satisfy the intellectual curiosity but will invite suggestions for further research which, if properly conducted, is bound to lead to valuable improvements in airplane design.

Aspect ratio which is so important in connection with the drag and the lift at a specific angle of attack has little, if any, relation to the large lift in the case of slotted wings. This fact simplifies the discussion to that of flow about a wing in two dimensions. For all ordinary wing sections the lift coefficient may be considered as increasing linearly with the effective angle of attack by about 0.1 per degree until the angle of attack of the chord has reached about 10°. With a further increase in angle of attack, there results a rapid change in the flow pattern, accompanied by an increase in the drag, the lift remaining the same or even decreasing.

It is of importance to see how this brief account of the facts underlying the theory of the unslotted wing must be altered to apply in the case of slotted wings. If the unslotted wing theory is applied to each component wing individually, first to the narrow front wing, hereafter called the slat, and then to the main wing in rear of the slat, some progress will be made towards explaining the larger lift caused by the slat. It will be found, however, that the total effect computed by this method is not large enough to agree with experiment.

First, let us consider whether it is possible for a slotted wing, when considered as a whole, to have a maximum lift coefficient larger than the maximum lift coefficient of its components. In other words, if both the slat and the main wing have maximum lift coefficients of say 1.5, based on their separate areas, the question is: can the composite wing have a maximum lift coefficient based on the total area of say 2.5? Obviously it cannot as long as all three lift coefficients, namely that of the slat, that of the main wing and that of the composite wing, are computed by the use of the same velocity, namely the velocity of flight. This raises the question whether use of another velocity, in the case of the slat, would not comply better with the wing-section theory, and whether the more correct velocities are sufficient to explain the larger lift.

The use of the local velocity, rather than the velocity of flight, is common practice in the older papers on biplane theory. The air velocity at each element of a composite wing should not be considered, according to the biplane theory, to be the velocity of flight but rather that which includes the interference components due to the adjacent wing or wings. This method should also be valid for the components of a composite wing.

First, consider the air force of the main wing which experiences the interference of the slat. It can be seen at once that the interference effect on the main wing cannot be large, since the narrow slat is unable to set up velocities extending far enough to influence materially the much larger main wing, and since the local velocities in the rear of the slat or of any wing are low. Hence, in the light of the wing-section theory, we cannot expect an appreciable increase of either the lift or the formal lift coefficient (computed with the velocity of flight) of the main wing. This is so obvious that it is not necessary to support it by mathematical detail. The application of the wing-section theory to the main wing disappoints the investigator as a

rational explanation of the entire lift increase as found experimentally.

The experimental results obtained by means of pressure distribution measurements by A. Ormerod,[1] for example, show likewise that "the lift coefficient of the main wing alone, operating behind the slat, never exceeds the maximum lift coefficient of the unslotted section" and that the slat bears the burden of the increase in lift. This confirms our contention.

The explanation of the slat lift, however, requires more study. In view of the relatively large chord of the main wing, the entire slat can be considered to be in the immediate neighborhood of the main wing, although the reverse does not hold. The slat is, moreover, located near the front or leading edge of the main wing where the local velocities which are set up by the main wing are known to be large. Let us form an estimate of the magnitude of these local velocities and then compare them with the interference velocities which are necessary to explain the large lift of the slat.

Let the velocity in front of a wing with an angle of attack of 30° be estimated for a position about 5 percent of the chord in front of the leading edge. This would correspond to the proportions in Ormerod's tests. Since an estimate of the velocity is sufficient for this explanation, one can proceed with a minimum of mathematical analysis. The amount of work involved is lessened not only by segregating the dominant physical effects from the non-dominant ones but also by segregating the dominant mathematical terms from the non-dominant terms. This approximate solution will enable one to compare the local and interference velocities and to explain the large lift on the slat; further separation would be wasted.

The analysis proceeds by substituting the nearest ellipse for the wing section and by substituting for the ellipse a straight line connecting the two foci. The component of the flow velocity normal to the straight line should not be noticeably different from the normal component of the velocity of flight, that is, $V \sin \alpha$. Now, in the plane of complex numbers, let the straight line extend between the two points $\pm i$. The velocity function of the flow set up by a lateral motion of this straight line is represented by the velocity function,

$$F_z = V \sin \alpha (1 - z/(1+z^2)^{\frac{1}{2}}),$$

where z denotes the complex variable representing the points of the plane. This function gives zero velocity at infinity, and between the foci points on the straight line, where z is $\pm i$, the second term in the bracket becomes purely imaginary and hence the real component becomes constant as required. Since the velocity distribution near the center, $z=0$, does not vary much, the same flow will also serve approximately for elongated ellipses with the ends of the straight line as foci. The value of z representing the region of interest is substituted in the expression for the velocity function. Now the distance of the center of the slat from the leading edge or from the focus is about 5 percent of the chord, since the distance between the leading edge and the adjacent focus can be neglected in the present case of a very elongated ellipse. We have, therefore, to substitute $z=1.1$ into our expression for the velocity function and 2 for the length of the straight line representing the chord. This gives an interference velocity of the order of magnitude.

$$V \sin \alpha (1 - 1.1i/(1-1.1^2)^{\frac{1}{2}}) = 0.76\, V \sin \alpha = 0.38\, V,$$

since the angle of attack is about 30°.

This, however, is only a portion of the interference, since we have not yet taken into account the circulation flow of the wing near the leading edge. The circulation interference, under ordinary conditions, is about equal to the interference of the transverse flow. In the present case, the circulation could correspond, at best, to an angle of attack of say 10°, because the lift of the main wing corresponds to that angle of attack. The total interference would then be of the order of magnitude of half the velocity of flight. It would be larger in rear of the slat, because the trailing edge is closer to the main wing and smaller at the leading edge of the slat; perhaps the resultant effect would correspond to a value greater than the interference at the center of the slat. The difference, however, would be small. It should be noted that the interference just mentioned must be superposed on the com-

[1] A. Ormerod, R. and M. No. 1477, p. 4, line 3.

ponent of the velocity of flight parallel to the wing section. This component, which is slightly smaller than the velocity of flight, is not parallel to the total interference velocity. Taking these facts into consideration, we are forced to the conclusion that the local velocity of the slat will, in no case, exceed one and one-half times the velocity of flight. Furthermore, the slat increases the apparent lift coefficient of the wing by about 0.7 when its chord is 14.5 percent of the chord of the main wing. Hence the lift coefficient referred to the area of the slat would be increased by 0.7/0.145, or approximately by 5, or say from 1.6 to 6.6, which is slightly more than a four-fold increase over its original value. Also, since the lift varies as the square of the velocity, the local velocity must be about twice the velocity of flight. The former considerations of local velocity explain, at best, half the lift increase.

There must then be a second effect not associated with single wings in free air. It cannot be a change of the direction or the curvature of the local air flow since these two influence only the lift at a given angle of attack and not the maximum lift obtainable. There remains then only the local rate of variation of the velocity, the gradient of its magnitude and the difference of direction of the velocity between adjacent points. As a rule, the rear edge of the slat is located in a region of rather high local or interference velocity, while the leading edge of the slat is in a region of much smaller local velocity. The air flows through the slot in a convergent manner. It is as if the slat were located in a tube of flow of narrowing cross section. It is, however, well known that convergent air motion encourages a smooth and orderly air flow. In front of an obstacle much less streamlining is required than in the rear, because the air is convergent in front and divergent in the rear. For the same reason, a venturi tube has to be longer aft the smallest cross section than in the front. It therefore stands to reason that the convergence of the local air flow along the slot is most probably responsible for the lift increase otherwise unexplained. The burbling of the fluid in the case of a wing with a slat is delayed in consequence of the convergence of the local flow. This is as far as reasoning will permit us to go. Whether or not the explanation is a correct one can only be proved by experiments. Wing sections should be investigated in convergent flow in this connection to see whether their maximum lift coefficient can be increased.

There is a second very interesting test suggested by the preceding analysis. A slotted wing is considered as the combination of a small wing and a large one, the small one being in the local field of the large one and under the influence of the increased velocity and convergence created by the large wing. This is similar to a multiple venturi tube where a small venturi is placed at the narrow cross section of the larger one in order to utilize its large local velocity. This method of increasing the vacuum has been carried further in this case, by placing a third tiny venturi into the second one at its smallest cross section. The analogy to the wing is obvious. It would be very instructive and interesting to place an even smaller slat near the front of the present slat in a local region of large velocity and convergence and to observe the resultant lift. As with the venturi tube, the principle cannot, of course, be carried too far. We cannot put one hundred venturi tubes one into the other, not only because that is not practical but also because the effect would, in the end, be lost. The pressure differences would assume magnitudes large enough to obtain compressibility effects. The velocities would approach the velocity of sound where the ordinary laws of a perfect and incompressible fluid no longer hold.

This analysis may be concluded with a few words about a quite different theory or explanation of the wing slot effect. L. Prandtl and A. Betz[2] explain the lift increase of the slot by its effect on the boundary layer. They say that the air stream flowing through the slot lifts or raises up the boundary layer. In other words, an attempt is made to explain the slot effect by direct air-friction influences, the argument being that by subdividing the wing the friction effect is divided. The main wing remains subject to the effects of its own surface friction but is no longer under the influence of the friction along the slat surface. On the contrary, the friction layer, which means the layer of air strongly

[2] L. Prandtl and A. Betz, *Ergebnisse der Aerodynamischen Versuchsanstalt zu Göttingen*, Lieferung 3, p. 9, line 17.

affected by the surface friction of the slat, passes by the main wing at a safe distance without unfavorably affecting the air flow.

This explanation seems unsatisfactory in several respects. According to it, an unslotted wing would not have less desirable characteristics than the same wing behind a slat, hence the latter wing should not have any larger lift than an unslotted wing, as indeed it has not, and we are exactly where we were before. Furthermore, since the effect of the slot on the main wing and not the effect on the slat is discussed, the explanation would imply that the main wing bears the burden of the lift increase, whereas the experiment shows that the slat does. Further, if the effect is a pure and direct friction effect, the same result for both cases would be obtained by diminishing the influence of the friction, for instance, by increasing the model size. Experience shows that the maximum lift coefficient is not increased by an increase of the Reynolds number but rather decreased in keeping with a much broader principle.

Finally, it would seem that an explanation based on the application of a generalized wing theory and applied to the components of the composite wing is preferable to an explanation based on direct friction effects. We do not know enough about air friction. The fundamental problem of the motion of slightly viscous fluids has not yet been solved. Any explanation based entirely on air friction can only explain one unknown quantity by another unknown quantity. This is not only logically unsatisfactory but also leaves the alleged explanation useless for the further advancement of science. A reduction of the problem to an application of a wing theory applied to each individual wing will lead directly to a chain of constructive suggestions for further theoretical and experimental research.

Comments on Meteorological Research

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ONE of the greatest obstacles to progress in meteorology is undoubtedly to be found in the wide gulf between the mathematical theory on one hand and the applied science, weather-map analysis and forecasting, on the other. In aerodynamics, it is possible to advance a theory and then to proceed to a set of specially arranged experiments from which the validity of the theory may be conclusively established. In meteorological research, experimental verification is usually impossible and one is therefore forced to study the full scale experiments provided by the atmosphere. In this study the chief difficulty lies in the number of variables involved in even the simplest phenomenon.

To eliminate all but one of these variables, two methods are possible. Utilizing a great number of observations, one may group the observed effect with respect to one selected variable, assuming the random distribution of the other elements to be the same in each group. This might be true if assurance could be had that the variables really are mutually independent but this, unfortunately, is seldom the case.

The other method consists in a study of individual cases, a procedure which has gained increased favor with the advent and ascendance of the Norwegian Meteorological School. The chief drawback to this method consists in the obvious opportunities it offers for a biased selection of evidence in favor of a preconceived theory.

It is no wonder, then, that meteorological theory in many cases has degenerated into pretty pieces of mathematical exhibition, where the postulates lack all resemblance to the conditions actually found in the air and where the results cannot be checked, because they belong to an imaginary world where all equations are linear, the atmosphere homogeneous and incompressible and the surface of the earth as flat as in pre-Ptolemeian days. This flight of meteorological theory from reality has caused such a deep distrust, particularly in the United States, of theoretical investigations that synoptic meteorologists have restricted themselves to an accumulation of weather-map experience which is seldom or never interpreted except in the most superficial sense. Education in even elementary classical physics is not considered necessary or even desirable. This development has gone so far that even well-established, simple physical principles, such as dynamic heating or cooling by vertical displacements, are often completely disregarded in the analysis of synoptic developments.

It would seem, then, that the principal task of any meteorological institution of education and research must be to bridge the gap between the mathematician and the practical man, that is, to make the weather man realize the value of a modest theoretical education and to induce the theoretical man to take an occasional glance at the weather map.

At this point it is important to emphasize that the purpose of meteorological theory for many years yet will be far more limited than that of a theory in, say, celestial mechanics, the object of which is to *predict* future events. In meteorology we are satisfied if the theories help us to understand the individual atmospheric phenomena which combine to make the weather. Numerical prediction on the basis of a reasonably complete physical theory is too far off to deserve immediate attention.

Because of this modest goal, the restrictions and postulates of theoretical meteorology often constitute a first step which is both permissible and desirable, provided they do not affect the essential character of the problem. However, such an intelligent adaptation of the problem is possible only through close cooperation between the mathematician and the synoptic meteorologist.

The polar front theory, developed by V. and J. Bjerknes, Solberg, and Bergeron, beyond a doubt represents the most successful effort as yet made to bridge the gulf that separates the two meteorological camps.

The introduction of air masses and fronts, conceptions based on the occurrence of large air currents of horizontally approximately homogeneous characteristics, separated by narrow transition zones of steep gradients and strong atmospheric activity, has given to the weather map a living reality which it entirely lacked in the days of moving isobars, *highs* and *lows*.

Although the experienced man often may recognize the fronts by a glance at the map this is not always the case. Often they are masked by friction and radiation influences near the ground. "False" stationary fronts may appear due to abrupt changes in the characteristics of the underlying surface (for instance, at a coast line). But even if we allow for the uncertainty or errors introduced due to the fact that our surface data are not always representative of conditions in the free atmosphere, there is such a difference between the analyses made in the early part of the last decade with those of the present day that one is led to the conclusion that the polar front theory is just as much a *method* of looking at the weather map as an actual statement of physical laws. For this reason it would be absurd to claim that the polar front theory does not apply west of the Rocky Mountains just because ideal cyclones are not generally found on the weather maps for this region. They cannot be found, partly because the network of stations is entirely too wide-meshed in the western part of the United States and partly because the fronts are rather diffuse in the surface layers on account of the almost identical oceanic trajectories of most of the participating air currents. The polar front theory does not state that ideal cyclones should be found along the Pacific Coast. It does say, however, that it is always possible to describe the structure and the behavior of extra-tropical cyclones in terms of air masses and fronts. Analyses made on airway meteorological material from the San Joaquin Valley and Southern California, a region which is far more sluggish meteorologically than the state of Washington or British Columbia, show that this is entirely feasible.

In the theoretical investigations by the Norwegian School the narrow transition zones have been supplanted by surfaces of discontinuity. Thus the theoretical problem resolves itself into a study of the dynamics of internal atmospheric boundaries. V. Bjerknes and Solberg have studied various types of wave motion in such boundaries. Though the restrictions introduced to facilitate the mathematical treatment are too great to permit an immediate acceptance of their findings as characteristic of the real atmosphere, they have obtained several results of definite practical significance, such as values for the rate of progress of these waves and a criterion for the stability of front surfaces of varying angles of incidence.

The existence of rapidly progressing waves of dimensions similar to those of extra-tropical cyclones was first demonstrated synoptically by Bergeron in 1924 as a result of a painstaking study of a certain European synoptic situation, with the aid of an enormous observation material. With the improved technique of front and air mass analysis it has become a simple matter to identify some of these waves. The daily working maps on file at the Massachusetts Institute of Technology contain a number of cases of quasi-stationary fronts with progressive waves, although these maps have been analyzed in much less detail on account of the limited time available.

The vertical structure of an air mass depends partly upon its source region, partly upon its trajectory. If these two elements are known, it is possible to predict, in a general way, the vertical distribution of temperature and humidity. It is possible to check these conclusions in a qualitative fashion through a study of the typical cloud forms associated with each air mass, since each type of temperature and humidity stratification has its characteristic cloud forms. A detailed check, however, is possible only from a study of pressure, temperature and humidity records obtained through airplane or kite soundings. Using a large amount of kite data from the Weather Bureau aerological stations my colleague, Dr. Willett, has recently completed an

investigation of the vertical structure of American air masses. The results of this work will be given in the next issue of this Journal. A preliminary study of the air masses of the Pacific based on airplane records from Honolulu, San Diego and Seattle, has been made at M.I.T. by Byers and will soon be published by the Scripps Institution in La Jolla. However, the amount of aerological material from the West Coast and from the North Pacific is too scanty for conclusive discussion.

Parallel with these synoptic investigations, theoretical studies have been made which have led to the development of a simple graphical method of identifying air masses with the aid of their most conservative properties, entropy of the dry air and specific humidity. An atmospheric sounding, if plotted with these two quantities as coordinates, gives a characteristic curve which, at least during the winter season, remains remarkably constant throughout the life history of the air mass.

Outstanding problems still awaiting their final solution in connection with the study of air mass properties are the following:

1. A determination of the rate at which an air mass from a given source region picks up moisture from the ground and warms up because of increased radiation heat income, coupled with increased turbulent or convective transport. The theoretical solution of this problem should be checked through a study of aerological ascents along the trajectory of the air mass.

2. A study of the rate of accumulation of energy available for convection and thunderstorms in stagnant air masses, coupled with a study of the mechanism through which this energy is released.

Both problems are of significance in connection with summertime forecasting and both require a large amount of aerological material for their solution. Such material is not now available and would require the organization of a small-mesh aerological network, each station making several soundings a day.

An aerological airplane station was established at Boston in November, 1931 and daily flights are made up to a level of approximately 5 km. The data from this station, combined with the soundings from the Weather Bureau aerological stations in Cleveland, Chicago and Omaha, all at about the same latitude and fairly evenly spaced, offer an excellent opportunity for a charting of the vertical structure of passing warm sectors and polar air wedges. A preliminary survey of the available observation material seems to indicate that in general our cold air masses are more shallow than those of Europe, a result which becomes understandable when one considers the great difference in latitude. Thus Scandinavian meteorologists, in discussing European conditions, sometimes dismiss the first kilometer as non-representative of the air mass, while in the United States, especially in the South and Southeast, a stagnant polar air mass sometimes does not reach higher than fifteen hundred meters.

During the last few years attention has been focussed upon the strong pressure and temperature changes at the base of the stratosphere. Bjerknes and Palmén interpret these variations as wholly or in part dynamic effects of the succession of cold and warm currents within the troposphere. On the other hand, Austrian meteorologists have claimed that the substratosphere must be regarded as an additional center and source of atmospheric activity. If the Scandinavian notion is correct, one would expect that the stratospheric variations in temperature and pressure over the United States would be less marked because of the fact that our wedges of polar air do not reach so high up.

The preceding remarks should suffice to demonstrate the fecundity of the Norwegian ideas as a working hypothesis. American participation in the research work which has sprung up with the polar front theory is still very slender and largely confined to a sterile discussion of the somewhat antiquated Norwegian conceptions of the early 'twenties. Vigorous American participation in research on current problems is desirable but must be preceded by general acceptance of the modern methods of front and air mass analysis.

Fluid Flow Near a Smooth Surface

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IT is known from model experiments that the air in contact with the surface of a streamline body moves with it, and that the regions of easy flow which surround the body are connected to the surface through a relatively thin layer of air, within which the velocity falls to zero. It is within this boundary layer that the resistance to motion is transmitted to the surface.

At speeds of flight, the flow in the boundary layer over the front part of a streamline body is regular, or as it is more often called, laminar, and the elements of fluid passing through any point in the layer behave in a simple and definite manner. At some distance beyond the nose of the body this regular motion breaks down, the movements of the elements become confused, and the flow is said to be turbulent.

Measurements made with small Pitot tubes and hot-wire meters have shown that the velocity in a turbulent boundary layer falls rapidly as the distance from the surface diminishes. Viscous forces then predominate, and it has been suggested that the flow simulates more and more closely a streamline character as the surface is approached. The intensity of surface friction would then be given by the product of the velocity gradient at the surface and the coefficient of viscosity. To determine whether this relation held, Stanton[1] made very careful measurements at the surface of a pipe in which the flow was turbulent. It was known that a reliable measurement of the velocity gradient at the surface could not be obtained with a Pitot tube of the usual type, even when the diameter of the mouth was very small, since a sufficiently close approach to the surface would not be possible: or with an exceedingly fine hot wire, because in addition to the forced convection due to the wind stream there would be a loss of heat by conduction across the thin layer of air between the hot wire and the surface. To overcome these difficulties, an ingenious surface tube (Fig. 1), which had for its inner wall the surface of the pipe itself, was designed and used by Stanton. Because of the extreme smallness of the tube, a special method of calibration had to be devised to determine the effective distance from the surface corresponding to the speed calculated from the pressure measured at the mouth. The product of the measured velocity gradient at the surface and the coefficient of viscosity of the air was found to be in fairly close agreement with the intensity of surface friction. More recently, the intensity of friction on the surface of an aerofoil*[2] has been determined at the National Physical Laboratory from measurements of velocity gradient taken with a particular form of Stanton tube constructed on the top of a circular rod designed to pass with a small clearance through holes drilled in the polished surface of a metal model. Three tubes were used. Photographs of the mouths of these tubes are given in Fig. 1. An interesting feature shown by the calibrations of these tubes is that the ratio of the effective distance to the width of opening increases as the width decreases. The effective distance of the smallest tube (No. 1) is in fact greater than the width of the opening (0.0020 inch), and the observations taken with this tube were not appreciably nearer the surface than those taken with the largest tube No. 3 (0.0044 inch). The distribution of frictional intensity was determined from measurements of velocity taken at 0.002 to 0.003 inch from the surface. The frictional drag of the aerofoil obtained from an integration of the frictional intensities was found to be in fairly

[1] Stanton, Marshall, Bryant, Proc. Roy. Soc. **A97** (1920).

* The intensity of the surface friction on a circular cylinder has also been measured in a similar manner. (R. and M. No. 1369.)

[2] Fage and Falkner, Proc. Roy. Soc. **A129** (1930).

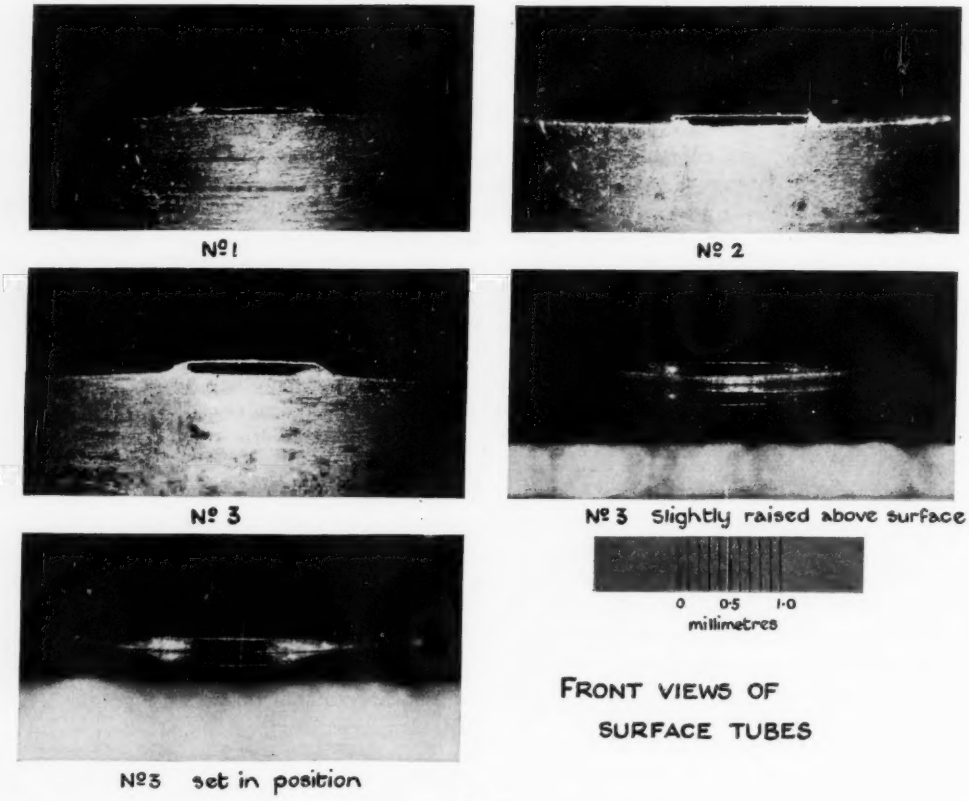

FIG. 1.

close agreement with predictions by two other methods.

The work with Stanton tubes leads then to the conclusion that in eddying flow the gradient of mean speed tends to a value at the surface which seems to be the same as that for a streamline motion having the same intensity of friction on the surface. Is then the flow in a surface layer of small but finite thickness of a streamline character? To obtain an answer it is necessary to watch the movements of particles very near the surface.

The scale of turbulence near a surface is small, and it is most important that the method of observation should not interfere in any manner with the internal motions of the fluid. The method of examining fluid motion with an ultramicroscope[3] which has been in use for some time at the National Physical Laboratory satisfies this condition, since neither extraneous particles nor measuring instruments are introduced into the fluid. The principle of the ultramicroscope depends on the fact that very small particles always present in fluids, such as air or tap water, but invisible in ordinary light even under the most powerful microscope, become visible when intensely illuminated and observed against a dark background. Even particles smaller than the wave-length of light become visible as bright points of light, if the intensity of the light beam is sufficiently great. Fig. 2 gives a photograph of the ultramicroscope arranged to examine turbulent flow in a square pipe, with 0.89 inch sides, a flow which is of the same kind as that in the turbulent boundary layer of a flat plate or a streamline body. A beam of intense bright light passing through a window in one side of the pipe illuminated particles contained

[3] Fage and Townend (with Appendix by Taylor), Proc. Roy. Soc **A130**, 656 (1932).

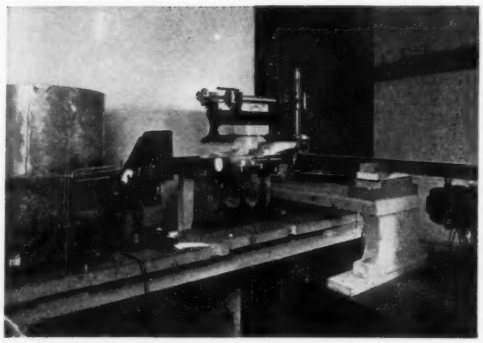

FIG. 2.

in the water, and the motions of these particles as they flowed downstream were viewed through an adjacent window with a microscope. When the motion was turbulent the particles passing through any fixed point moved in different directions, and the short traces of the paths seen in the microscope appeared as bright rectilinear streaks inclined at various angles to the mean direction of flow. The maximum obliquities of these streaks were measured. Measurements were also made of the mean speed and the maximum velocity fluctuations in the axial direction, by a technique which was based on the fact that illuminated particles were brought to apparent rest by motion of the objective of the microscope at the appropriate speed.

It was found that the maximum obliquity of particles in an axial plane normal to a wall of the pipe increased slowly with the distance from the axis until a maximum value was reached about 0.05 inch from the wall. The value then fell steadily to zero at the wall, so that particles seen when looking along and very close to the wall appeared to be moving in laminae parallel to the wall. The maximum obliquity in planes parallel to the wall increased progressively as the wall was approached and at the small distance of about 0.003 inch the large angle of 70° was observed. On viewing the surface with a microscope having a very high magnification (200) illuminated particles within a surface layer of thickness 0.001 inch were brought into focus. The faster moving particles in this layer were obviously moving in sinuous paths, and careful observation showed that the slower ones, some of which were estimated to be not more than 2.5×10^{-5} inch from the surface, crossed and recrossed several times a hairline in the eyepiece of the microscope. Occasionally, when a group of the faster particles moved laterally sideways the slower particles would also shift laterally. Also, the axial motions of the slower particles were often jerky.

Fairly conclusive evidence was therefore obtained to show that when the general motion is turbulent the flow in a very thin surface layer is not regular, although the friction on the surface can be determined fairly closely from the relation $f=\mu(\partial V/\partial y)_{y=0}$, where V is the mean velocity at a distance y from the surface, and the value of $(\partial V/\partial y)_{y=0}$ is predicted from values of V measured with a small surface tube.

The ultramicroscope has also been used to observe the nature of the eddying wake behind long cylindrical bodies.[4] This work leads to the interesting conclusion that the disturbances in the eddying wake behind any long cylindrical body, bluff or streamline, placed with its length normal to the direction of flow, will be three-dimensional, under conditions of flight, even although the mean flow in planes at right angles to the length tends to be two dimensional. It is perhaps of interest to mention that photographs have recently been taken of some interesting views of fluid flow revealed with an ultramicroscope.

[4] Fage, A.R.C. (R. and M. No. 1510).

Stresses in Wire Wheels

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DURING the last two or three years the writer has been carrying out a research into the stresses caused in a wire wheel under loads applied to the rim. This work has been done for the Aeronautical Research Committee and the results have been published in a series of papers which are enumerated at the end of this note which gives a brief resumé of the problem, method of attack and certain results.

A wire wheel consists of a comparatively rigid hub and a rim connected to it by a large number of small section spokes which may be either radial or inclined to the radius. Since the flexural rigidity of the spokes is small it is sufficiently accurate for analytical purposes to assume that they are pin jointed to the hub and rim. With this assumption it is, of course, impossible for the radially spoked wheel to resist actions on the rim which are tangential, e.g., those due to the application of brakes: for efficient resistance to such forces the spokes must be inclined in the plane of the rim. To resist side forces on the rim the spokes must also have an inclination in a plane perpendicular to that of the rim.

The problem to be considered, therefore, was that of a wheel having radial or inclined spokes, held at the hub and subjected to loads at the rim acting radially, tangentially and transversely. An attack was first made by means of a strain energy treatment but the resulting equations were too complex for a complete solution to be obtained. This analysis did, however, yield certain important relationships between the resultant actions which were required. A second line of attack was suggested to the author by Professor R. V. Southwell, which was instrumental in completing the solution of the general problem.

It was assumed that the spokes could be replaced by a continuous disk which had the peculiar property of transmitting load in one direction only. If the spokes were radial, the disk could have stresses only in a radial direction; if they were inclined, the disk transmitted stresses only along the inclined lines. If more than one system of spokes were used in the wheel, as is commonly the case, each system of spokes was assumed to be replaced by an appropriate disk. The displacements of the rim under load, at any point ψ from an axis of reference are denoted by u and v in the radial and tangential directions, respectively. R is the radius to the centroid of the rim, R_0, the radius to the tangent circle on the hub, I, the moment of inertia of the cross section of the rim about its axis of bending, a, the cross-sectional area of the rim, E_1, the modulus of elasticity of the rim material, E_2, the modulus of elasticity of the disk material, t_1, the inward pull per unit length of rim of the disk acting at ϵ to the radius, t_2, the inward pull per unit length of rim of the disk acting at $-\epsilon$ to the radius, where ϵ is the angle of inclination of the spokes to the radii in the plane of the wheel. If the thickness of each disk at the rim is Δ, we have

$$t_1=(E_2\Delta/R)(u-v\tan\epsilon);$$

$$t_2=(E_2\Delta/R)(u+v\tan\epsilon).$$

Also let H be the tangential force on the rim at ψ, V the radial force and M the bending moment. Then

$$H=-(E_1a/R)(u+\partial v/\partial\psi) \qquad (1)$$

$$M=(E_1I_1/R^2)(u+\partial^2u/\partial\psi^2). \qquad (2)$$

The differential equation for rim displacements is then

$$\partial^6u/\partial\psi^6+2\partial^4u/\partial\psi^4+(1+K_1\cos\epsilon)(\partial^2u/\partial\psi^2)-K_1u\sin\epsilon\tan\epsilon=0,$$

where $K_1=2\Delta E_2R^3/E_1I$. The complete solution of this equation is

$$u=A\sinh\gamma\psi+B\cos\alpha\psi\sinh\beta\psi+C\sin\alpha\psi\cosh\beta\psi+D\cosh\gamma\psi+E\cos\alpha\psi\cosh\beta\psi+F\sin\alpha\psi\sinh\beta\psi,$$

where

$$\alpha=\left(\frac{\left(\frac{X^2}{4}+\frac{2X}{3}+\frac{4}{9}+\frac{3Y^2}{4}\right)^{\frac{1}{2}}+\left(\frac{X}{2}+\frac{2}{3}\right)}{2}\right)^{\frac{1}{2}},$$

$$\beta=\left(\frac{\left(\frac{X^2}{4}+\frac{2X}{3}+\frac{4}{9}+\frac{3Y^2}{4}\right)^{\frac{1}{2}}-\left(\frac{X}{2}+\frac{2}{3}\right)}{2}\right)^{\frac{1}{2}},$$

$$\gamma=(X-2/3)^{\frac{1}{2}},$$

$$X=(M+Q)^{\frac{1}{2}}+(M-Q)^{\frac{1}{2}},$$

$$Y=(M+Q)^{\frac{1}{2}}-(M-Q)^{\frac{1}{2}},$$

$$M=\frac{1}{27}+\frac{K_1\sec\epsilon}{6}(2+\sin^2\epsilon),$$

$$Q=\left(\frac{K_1}{27}[K_1^2\cos^3\epsilon+K_1\{2+\sin^2\epsilon(7+(27/4)\tan^2\epsilon)\}+\sec\epsilon]\right)^{\frac{1}{2}}.$$

Since for inextensional strain of the rim

$$u=-\partial v/\partial\psi,$$

a complete solution of the displacements of the rim is found and hence the forces t_1 and t_2 at any point are known. In the limit when $\epsilon=0$, γ is zero, and α and β reduce to the values for the radially spoked wheel, *viz.*:

$$\alpha_0=(\tfrac{1}{2}\{(K_1+1)^{\frac{1}{2}}+1\})^{\frac{1}{2}};\quad \beta_0=(\tfrac{1}{2}\{(K_1+1)^{\frac{1}{2}}-1\})^{\frac{1}{2}}.$$

The constants A, B, C, D, E and F are evaluated from the known conditions of the problem, and the work, although somewhat lengthy, is not difficult. Having found t_1 and t_2 for the disk, the load in a spoke of the actual wheel is determined approximately by integrating these stresses over a segment of the disk replacing the spoke considered. It will be apparent that the greater the number of spokes in the actual wheel the better will be the approximation.

In order to check the accuracy of the solutions thus obtained, a series of experimental wheels were constructed. The rims, four in number, of different sizes, were cut out of mild steel plate and the spokes were made of 4 B.A. aeroplane tie rods and $\frac{1}{4}''$ rods. Each rim was used with three, four, six, twelve and twenty-four spokes of each size and the loads in the spokes were measured by accurate extensometers for comparison with the calculated values. From these measurements the values of the bending moment in the rim could be deduced from the strain energy analysis and could thus be compared with the more approximate solution.

The general agreement between calculated and measured values of loads and rim actions was excellent and the experiments showed conclusively that the approximate solutions obtained were quite satisfactory for any wheel with more than 30 spokes. The formulae deduced for spoke loads, etc., were rather complicated and in view of the good experimental results obtained, these formulae were further simplified for convenience of calculation with very little loss of accuracy. The simplified formulae are as follows: Let

P be the inward radial load on the rim,

T_N, the maximum compression in any spoke,

T_T, the maximum tension in any spoke,

$m_0'R$, the maximum bending moment in rim.

Then $T_N/P=1-e^{-\beta_0\theta}$;

$$T_T/P=(1+\beta_0\pi e^{-\beta_0\pi/\alpha_0})/N;\quad m_0'/P=\beta_0/2(K_1)^{\frac{1}{2}}.$$

These results are only applicable to the case of the wheel with radial spokes. When the spokes are inclined the results are too complicated for inclusion in this note but in the original papers cited below full particulars, together with curves for rapid estimation, are given.

In dealing with the case of transverse loads on the rim certain assumptions were made to reduce the problem to manageable proportions and it was found that by a simple transformation of some of the quantities involved the stresses in the spokes and actions in the rim could be determined from curves already plotted for the radial and tangential cases of loading.

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3. Pippard, Francis and White, *Stresses in a wire wheel with non-radial spokes under rim loads*, R. and M. No. 1440; Pippard and White, Phil. Mag. **14**, 209 (1932); and Pippard and Francis, Phil. Mag. **14**, 436 (1932).

Effects of Compressibility on High Speed Flight

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It has been known that the air forces exerted on aerodynamic bodies follow the so-called "square law" except for the effects of scale and compressibility. The effects of dynamic scale, or Reynolds number, have become rather well known during the last 10 years, largely because of research conducted in the N.A.C.A. variable-density wind tunnel. The effects of compressibility have commonly been neglected because until the relatively recent development of the last Schneider trophy aircraft the speeds have been low as compared with the velocity of sound, and the consequent local pressures over the surfaces of high speed airplanes have differed but slightly from atmospheric pressure. At the present time, however, the speeds associated with the fastest airplanes approach 60 percent of the velocity of sound, and the induced velocities over their exposed surfaces lead to local pressures that differ appreciably from the pressure of the atmosphere. When this condition exists air can no longer be regarded as an incompressible medium. The effects of compressibility on the aerodynamic characteristics of airfoils have been under investigation by the N.A.C.A. in the high speed wind tunnel, and it is the purpose of this paper to examine the possibility of further increases in speeds in the light of this relatively recent research.

The following computations are made for a hypothetical airplane which, however, is not beyond the limits of possibility. The airplane is a scaled-up model which has been tested in the variable-density wind tunnel as part of an extensive investigation of wing-fuselage interference, and represents one of the best wing-fuselage combinations thus far produced. The geometric scale factor applied to the model dimensions was determined from the ratio of the model fuselage diameter to the diameter of a fuselage sufficiently large to house a 2300-horsepower Rolls-Royce *R* type engine. The model from which the airplane is derived is a mid-wing cantilever monoplane having a fuselage of circular cross section, and a straight tapered wing having the N.A.C.A. 0018 section at the center and the N.A.C.A. 0009 section at the tip. The principal full-scale airplane dimensions are given in Table I. An outline drawing of the airplane for which computations will be made is given in Fig. 1.

Table I.

Fuselage diameter	40 in.
Wing span	29.1 ft.
Wing area	141.2 sq.ft.
Wing chord (average)	4.85 ft.
Aspect ratio	6

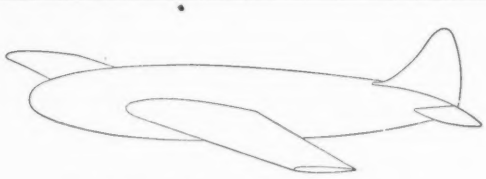

Fig. 1. Hypothetical high speed airplane.

The model, without tail surfaces, tested in the variable-density wind tunnel at a Reynolds number of 3,000,000, was found to have a minimum drag coefficient of 0.0115. If 10 percent of this value be added to account for tail-surface drag the resulting coefficient is 0.0127. No drag allowances need, or should be, considered for landing gear, cooling systems, or other appendages. The airplane should have completely retractable landing gear or be catapulted and landed on water with the aid of suitable retractable hydrodynamic surfaces. The surfaces of the airplane must be free from protuberances or excrescences because of the large amount of power required at very high speeds to overcome the drag of even the slightest surface irregularity. The engine will be completely enclosed and cooling can be provided by skin-type radiators.

Vision for the pilot can be obtained by transparent fuselage sections or indirectly by mirror systems. Inasmuch as the condition of the airplane herein assumed is not impossible of attainment, the airplane may be considered as an example of a high degree of aerodynamic refinement but not as an "ideal" airplane.

If the effects of compressibility are, for the moment, neglected, the computation of the maximum speed when a 2300-horsepower engine is used becomes relatively simple. The drag coefficient can be considered as less than the variable-density wind-tunnel value because of the higher Reynolds number of the airplane. The minimum profile drag coefficient may be reasonably estimated as

$$C_{D_0}\,\min = C_{D_0}\,\min(\text{tunnel})\left(\frac{\text{R.N. tunnel}}{\text{R.N. flight}}\right)^{0.12}.$$

This assumed reduction in profile drag is based on the results of tests on airship and airfoil models over a wide scale range, and is probably not greatly in error. In order to estimate the full-scale drag coefficient a speed of 550 miles per hour is assumed and the corresponding Reynolds number is computed. The full-scale minimum profile-drag coefficient then becomes, according to the preceding formula, 0.0098. A reasonable assumption regarding the probable propeller efficiency must next be made. The propeller pitch will exceed the optimum value, but the decrease of efficiency from this source will probably not be very great. The tip speed will be high but losses from this source may be kept to a minimum by suitable gearing and a reduced propeller diameter. The large forward thrust of the propeller will necessitate wide blades and probably a multibladed propeller. There are, of course, no direct experimental data on which to base an accurate estimate of the efficiency for such propellers, but it is believed that efficiencies at least as great as 75 percent may be obtained.

Assuming, then, 75 percent for propeller efficiency the maximum speed of the airplane, neglecting the induced drag, becomes

$$V_{\max} = (2\ \text{hp.}\ \eta 550/C_{D_0}\ \min\ fA)^{\frac{1}{3}} = 830\ \text{ft./sec.}$$

or

$$V_{\max} = 566\ \text{miles per hour.}$$

The slight error in the assumed Reynolds number from which C_{D_0} min was computed is negligible and within the probable error of the assumed effect of scale. The induced drag will cause but slight reduction in speed. Assuming 2.5 pounds per horsepower, the weight of the airplane is 5750 pounds and the lift coefficient at which the high speed flight occurs is approximately 0.05. The induced-drag coefficient, C_{D_i}, is therefore 0.0001, approximately 1 percent of the total drag.

It is interesting, now, to compute the landing speed of the airplane. The maximum lift coefficient as determined by tests in the variable-density wind tunnel is 1.516. The landing speed computed from this value is therefore

$$V = (2L/\rho C_L A)^{\frac{1}{2}} = 150.6\ \text{ft./sec.}$$

or

$$V = 103\ \text{miles per hour.}$$

The effects of compressibility as indicated from airfoil tests in the high speed tunnel will materially affect this calculated performance. Recent tests[1] demonstrate that the effects of compressibility are of considerable importance for speeds above 40 percent of the velocity of sound (306 m.p.h.). This result has been corroborated by tests on a series of symmetrical airfoils, the results of which are now being prepared for publication. The variation of the minimum drag coefficient with speed for a symmetrical airfoil, the N.A.C.A. 0012, is shown in Fig. 2. The drag coefficient is given as $D/\frac{1}{2}\rho V^2 A$. The Reynolds number for these tests is, however, much lower than that for the airplane under discussion because of the small size of the model. Unfortunately, the effects of large Reynolds number differences at these high speeds are unknown. These data, however, represent the best available information and will be applied without scale correction.

The landing speed of the airplane may be expected to be unaffected by compressibility because at such low speeds (103 m.p.h.) no measurable compressibility effect on maximum lift has been observed. It may therefore be

[1] John Stack, *The N.A.C.A. High-Speed Wind Tunnel and Tests of Six Propeller Sections*, Tech. Report No. 463, N.A.C.A., 1933.

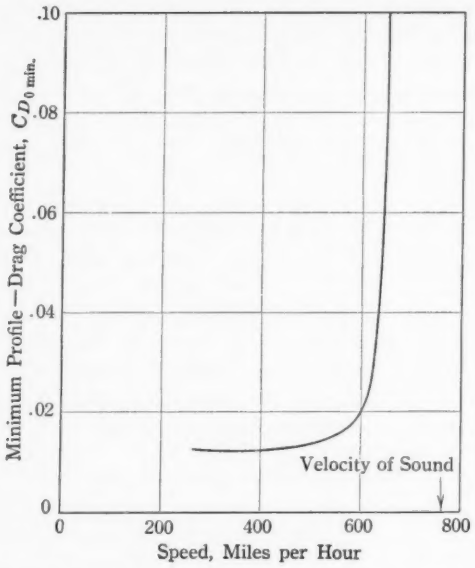

FIG. 2. Effect of compressibility on minimum profile-drag coefficient. N.A.C.A. 0012 airfoil.

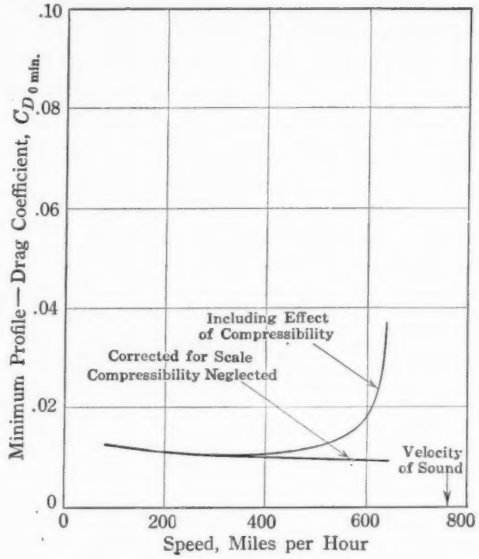

FIG. 3. Minimum profile-drag coefficient for airplanes.

taken that the landing speed of the airplane is not beyond reason.

The high speed of the airplane will, however, be materially reduced because of the effects of air compressibility. These effects have been estimated from the results of tests of the N.A.C.A. 0006, 0009, and 0012 airfoils in the high speed wind tunnel. In this estimation the drag coefficient of the airplane, 0.0127, was corrected by the previously given formula for scale effect to a Reynolds number corresponding to 306 m.p.h. Above this speed the drag coefficient was corrected in accordance with the high speed tunnel data. The resulting minimum-drag curve for the airplane is shown in Fig. 3.

A direct computation of the maximum speed cannot be made because of the C_D, variation with speed. An estimate could be made by a series of successive approximations but a better method which shows the compressibility effects in terms of horsepower differences for a given speed or in terms of speed for a given horsepower is to compute the horsepower-required curves. In this computation the induced drag and the additional profile drag due to lift have been added to the minimum profile drag of the wing. The weight has been held constant at 5750 pounds. The power curves are shown plotted in Fig. 4.

Assuming, as before, a propeller efficiency of 75 percent and 2300 horsepower, the speed loss due to the compressibility effect is shown by curves *A* and *B* as 39 m.p.h. Expressed in terms of horsepower at 524 m.p.h., the maximum speed corrected for compressibility, the effect is 325 horsepower, sufficient power to propel the airplane at 300 m.p.h.

The increasing importance of the compressibility effects has been foreseen by several research agencies, but the largest amount of work has been done by the National Advisory Committee for Aeronautics. Early tests[2] indicated the trend of aerodynamic characteristics with speed and were followed by improved means for studying the phenomena.

Some results are already available to designers in the form of new wing sections developed in the high speed wind tunnel. The drag of one of the new sections, the N.A.C.A. 204, is considerably lower than the drag for the section of our tapered wing having the same thickness but,

[2] L. J. Briggs, G. F. Hull and H. L. Dryden, *Aerodynamic Characteristics of Airfoils at High Speeds*, Tech. Report No. 207, N.A.C.A., 1925.

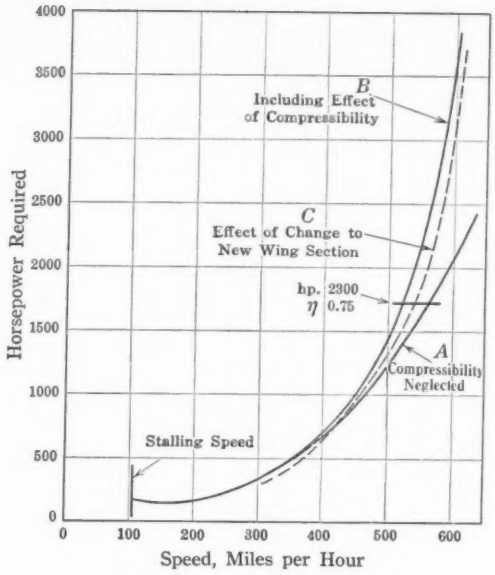

FIG. 4. Horsepower = required curves.

most important for purposes of high speed flight, the speed at which the compressibility burble of the N.A.C.A. 204 occurs (evidenced by the rapid rise of the drag coefficient at high speeds) is delayed to higher speeds. Unfortunately, no data are as yet available on the maximum lift and the effects of thickness changes for the N.A.C.A. 204. However, making reasonable assumptions regarding the effects of thickness variations and applying the indicated drag reduction to the parts of the airplane affected by the change to the new wing section, the high speed portion of a new power required has been computed and plotted as curve *C* in Fig. 4.

A comparison of curve *C* with curves *A* and *B* indicates a decided improvement at the lower speeds, due, of course, to the lower drag of the new section. A considerable increase in the maximum speed is also indicated. For the assumed power and propeller efficiency the maximum speed of the airplane is increased from 524 to 544 m.p.h. In terms of horsepower at 544 m.p.h., a direct saving of 315 horsepower is indicated. The improvement at the extremely high speed can be considered as due to the delayed compressibility burble.

It is apparent from the foregoing computations that much higher speeds than those so far attained are possible and likely. The attainment of these higher speeds is, however, dependent to a large extent on the effects of compressibility, first because of the compressibility effect on the airplane drag, and second because of the compressibility effect on the propeller efficiency. The aerodynamic characteristics of propellers suitable for such high speeds must also be investigated. The range of pitches for which tests have already been made must be extended and the effects of wide variations in blade shape and number of blades must also be studied.

The study of the compressibility phenomena is of relatively recent origin and much remains to be done. Practically nothing is known of the combined effects of scale and compressibility at very high speeds, nor is anything known of aerodynamic interference at very high speeds. The success so far achieved, however, is encouraging and it is reasonable to expect further aerodynamic improvement as the knowledge and understanding of the nature of the compressibility phenomena expand.

Speed of Air Driven Rotors in Gyroscopic Instruments

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THE importance of blind flying instruments is now generally accepted and gyroscopic instruments are used very widely in this connection. Although European design has employed electrically driven as well as air driven rotors, it is almost without exception the practice in this country to use the air jet drive which, by virtue of its simplicity in the instrument itself, is very desirable and reliable. The usual arrangement of such a system is to keep the whole inside of the instrument casing at a reduced pressure thus causing the differential pressure between the atmosphere and the reduced pressure in the casing to force air through the jet to impinge on the rotor buckets.

The speed at which the rotor is run in these instruments is of great importance. Excessive rotor speed may prove detrimental to the bearings while, if the speed of the rotor drops too far below its normal value, the nutations of the gyroscope begin to be troublesome and the instrument tends to become generally sluggish.

It is the purpose in the following simple considerations to develop an expression which will predict the rotor speed at various atmospheric conditions and thus anticipate the performance of the instrument at those conditions for the particular suction value available.

The rotor speed will depend on the suction on the instrument and also on the conditions of temperature and pressure in the atmosphere. Then when the rotor is running steadily,

The Jet Impulse Torque = Windage Resistance Torque + Bearing Friction Resistance Torque.

At the high speeds at which these rotors run the bearing frictional torque will be much the smaller of these two factors, the windage or air resistance playing the predominant part.

Considering first, therefore, the drop in rotational speed due to windage, if n_{drop} is the drop in revolutions per sec.,

$$n_{\text{drop}} = \text{a function } (\rho, n, r, \mu),$$

where ρ is the air density, n is the rotational speed, r is the effective radius and μ is the air viscosity.

Expressing n_{drop} in a power series,

$$n_{\text{drop}} = C\rho^{\alpha}n^{\beta}r^{\gamma}\mu^{\delta} + C_1\rho^{\alpha_1}n^{\beta_1}r^{\gamma_1}\mu^{\delta_1} + \text{etc.}$$

Considering this for dimensional homogeneity, the expression

$$n_{\text{drop}} = Bn \times \text{function of } (\mu/\rho n r^2) = Bn\phi(R_n),$$

where B is a constant, ϕ is some function, and $R_n = \rho n r^2/\mu$ = Reynolds number.

If there were no resistance, the rotational speed would be equal to $u/2\pi r$, where u is the velocity through the jet.

The drop in revs., therefore

$$u/2\pi r - n = Bn\phi(R_n) + \text{drop in revs. due to bearing frict.}$$

$$n[1 + B\phi(R_n)] = u/2\pi r - \text{drop in revs. due to bearing frict.}$$

Now considering bearing friction, suppose there is a constant frictional torque, Fr; then for steady rotation (considering bearing friction only)

$$C_2\rho u^2 r = Fr,$$

$C_2\rho u^2$ representing the effective momentum of the jet. This whole term will be smaller than the windage term in the complete expression and therefore very little error will be introduced by writing

$$C_3\rho n^2 r^2 = F$$
$$\therefore \quad n = K_1/\rho^{\frac{1}{2}}$$

(assumption is that u is proportional as a first approximation to n). n_{drop} due to bearing friction may, therefore, be written

$$n_{\text{drop frict.}} = K/\rho^{\frac{1}{2}}.$$

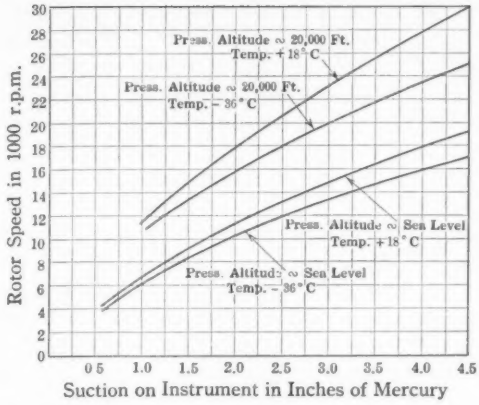

Press. Altitude ∞ 20,000 Ft.
Temp. +18° C
Press. Altitude ∞ 20,000 Ft.
Temp. −36° C
Press. Altitude ∞ Sea Level
Temp. +18° C
Press. Altitude ∞ Sea Level
Temp. −36° C
Rotor Speed in 1000 r.p.m.
Suction on Instrument in Inches of Mercury

FIG. 1. Sperry Artificial Horizon. Rotor speeds at various air pressures under various altitudes and temperatures.

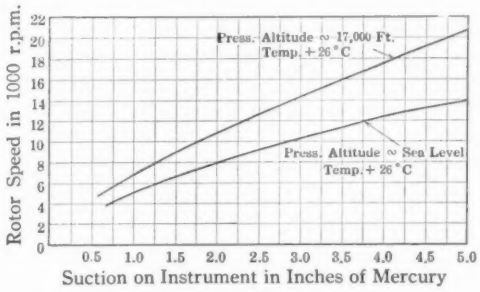

Press. Altitude ∞ 17,000 Ft.
Temp. +26° C
Press. Altitude ∞ Sea Level
Temp. +26° C
Rotor Speed in 1000 r.p.m.
Suction on Instrument in Inches of Mercury

FIG. 2. Sperry Directional Gyro. Rotor speeds at various air pressures at two different pressure altitudes.

The complete expression is then

$$n[1+B\phi(R_n)]=u/2\pi r-K/\rho^{\frac{1}{2}}$$

and when the differential pressure is Δp through the nozzle, then $u=\alpha(2\Delta p/\rho)^{\frac{1}{2}}$.

In this formula for the rotor speed, therefore, only α, K and $B\phi(R_n)$ have to be evaluated.

α is simply the orifice coefficient of the nozzle through which the air issues to impinge on the rotor. α is a function of Reynolds number but its variation is small enough to be neglected; α has been assumed here $=0.95$.

K is the constant containing the bearing friction effect and may be evaluated in two ways. It is evident that when $n=$ zero

$$K=(\alpha/2\pi r)(2\Delta p)^{\frac{1}{2}}.$$

Thus by finding the differential pressure at which the rotor stops K can be found.

Alternatively, if two curves are available of n *vs.* Δp for two different values of ρ, then two points having the same value of R_n can be picked. If the conditions at these points be $n_1\Delta p_1\rho_1$ and $n_2\Delta p_2$ and ρ_2

$$[1+B\phi(R_n)]=\frac{\alpha}{2\pi r n_1}\left(\frac{2\Delta p_1}{\rho_1}\right)^{\frac{1}{2}}-\frac{K}{n_1(\rho_1)^{\frac{1}{2}}}$$

$$=\frac{\alpha}{2\pi r n_2}\left(\frac{2\Delta p_2}{\rho_2}\right)^{\frac{1}{2}}-\frac{K}{n_2(\rho_2)^{\frac{1}{2}}}.$$

This may be solved for K. When K is determined, from the curves of n *vs.* Δp values of $B\phi(R_n)$ can be calculated for various values of R_n.

Whereas K will vary from instrument to instrument of the same type because of slight variations in the bearings, it is evident that $B\phi(R_n)$ will be substantially the same for all instruments of one type. Figs. 1 and 2 show the

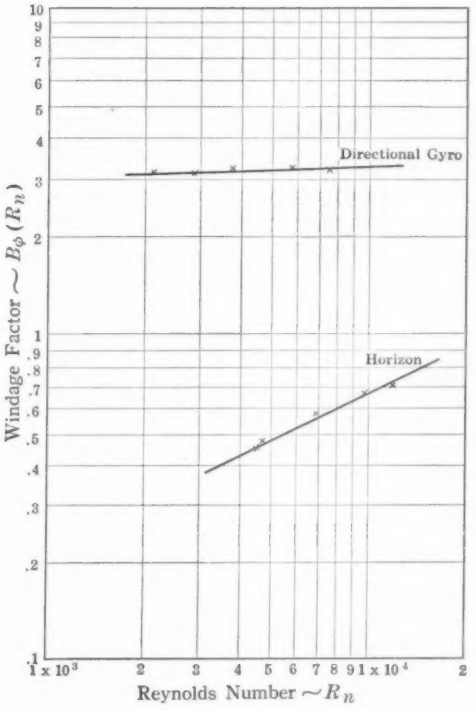
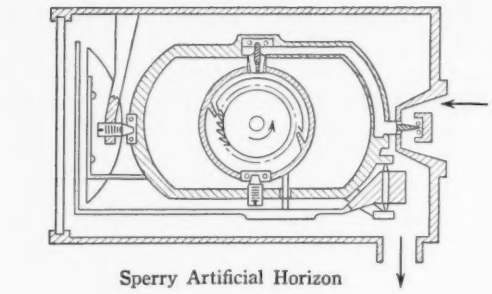
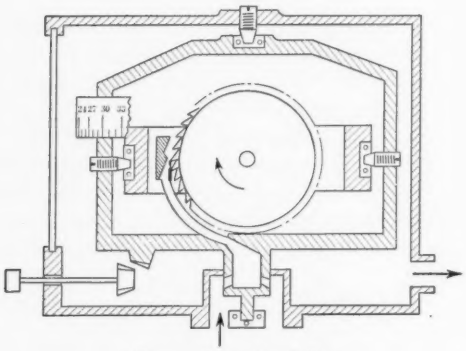


FIG. 3. Variation of windage factor as a function of Reynolds number. Sperry Directional Gyro and Artificial Horizon.



Sperry Artificial Horizon



Sperry Directional Gyro

FIG. 4.

variation of rotor speed with suction for various conditions of pressure and temperature for the Sperry Artificial Horizon and Directional Gyro, respectively. The values of $B\phi(R_n)$ calculated from these curves are shown plotted against (R_n) in Fig. 3.

Examination of Fig. 3 shows that the windage factor may be expressed over the practical range as $B(R_n)^\delta$ where δ is the gradient of the line on the logarithmic plot; also that the windage factor for the Directional Gyro is much greater than that of the Horizon and that δ is small for the former instrument. If δ were zero, then it is evident that windage loss would be entirely turbulent; thus the value of δ indicates the degree to which viscosity is entering into the resistance effect.

δ is, therefore, greater for the Artificial Horizon because the rotor is completely enclosed in a casing with small clearances. This casing also has the effect of reducing the total value of $B_{\text{Hor.}}(R_n)^\delta$ for the Horizon as it reduces the size of the turbulent eddies. In the Directional Gyro the rotor is open and the turbulent eddies are large, viscosity enters less, and, therefore, δ is small and the value of $B_{\text{Dir.}}(R_n)^\delta$ is much greater. This indicates well the beneficial effect of encasing the rotor in such an instrument.

The above analysis permits the separation of the various factors affecting the rotor speed and it then becomes evident that with an increase of altitude with a constant suction on the instrument, the decreased pressure will cause the rotor to speed up while the decrease in temperature may cause K to increase and thus tend to slow the rotor down; but with carefully designed bearings the net result is an increase of rotor speed with altitude.

REFERENCES

1. M. D. Hersey, *General Classification of Instruments and Problems*, N.A.C.A. Tech. Report No. 125.
2. *Direction Instruments*, N.A.C.A. Tech. Report No. 128.
3. E. Buckingham, *Windage Resistance of Steam Turbine Wheels*, Bur. Standards Papers No. 208, 1914.
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5. H. N. Eaton, *Aircraft Instruments*.

Institute of the Aeronautical Sciences

Aims

THE Institute of the Aeronautical Sciences was organized to promote the application of science in the development of aircraft. By grading members according to their qualifications, recognition is given to outstanding professional work and an incentive is provided younger members for the investigation of new aeronautical problems. The presentation and discussion of technical papers at Institute meetings afford members opportunities to hear and meet the leaders in specialized fields of the aeronautical sciences. Through the publication of the Journal of the Aeronautical Sciences, members are provided with an opportunity to publish and to read new knowledge and new applications in the field of aeronautical endeavor. By including in its membership the leading aeronautical specialists in other countries, an interchange of international aeronautical scientific progress is made possible. The organizers of the Institute believe that the future development of aircraft depends on scientific research and engineering experiment applied to design. As practically every scientist and engineer already belongs to his own special professional society, in order to bring together all specialists contributing to aeronautical progress, the dues of the Institute have been made nominal. Through the foregoing aims, as well as others too obvious to specify, the Institute is providing the United States with a scientific society similar to those which have had such a beneficial influence on aeronautical progress in other countries.

History

The Institute of the Aeronautical Sciences was incorporated under the membership corporation law of New York State on October 15, 1932. The purpose of the Institute as stated in the Charter is as follows:

"To advance the art and science of aeronautics; to publish works of literature, science and art for such purpose; to do all and every act necessary, suitable and proper for the accomplishment of any of the purposes or the attainment of any of the objects or for the furtherance of any of the powers hereinbefore set forth, either alone or in association with other corporations, firms or individuals and to do every other act, thing or things, incidental or appurtenant to or growing out of or connected with the aforesaid science or art, or powers or any part or parts thereof, provided the same be not inconsistent with the laws under which this corporation is organized or prohibited by the laws of the State of New York."

The incorporators at the first meeting, on October 17th, 1932, adopted a constitution for the Institute which gives broad powers to the Council until such times as the members of the Institute wish to make changes. On October 24th, the Council adopted By-Laws and elected temporary officers to serve until the first annual meeting of the Institute which was held on January 26th, 1933.

At the Founders' Meeting held at Columbia University on January 26th, 1933, the Constitution and By-Laws were approved by the members and the temporary Council was re-elected. The papers presented at the Founders' Meeting were published and bound copies were distributed to all members and to aero clubs, universities, libraries and aviation officials in many countries.

A members' clubroom and an office have been provided, by generous donors, at Rockefeller

Center, New York, for the social and administrative activities of the Institute. A library of books, magazines and pamphlets is now available to members.

The need for such an aeronautical organization is best indicated by the membership list which includes leaders of aeronautical thought in the United States and abroad.

Publications

The Journal of the Aeronautical Sciences is the official publication of the Institute. It is published in collaboration with the American Institute of Physics under the direction of an editorial board composed of members of the Institute. It will be the policy of the Journal to print contributions of scientific and technical interest to members. In general, it will provide an opportunity for members to publish articles disclosing new knowledge and new applications in the field of the aeronautical sciences. The Journal will print a few complete professional papers in each issue and a larger number of shorter notes and critical comments. The Journal will restrict itself to the aeronautical aspect of advances in science and engineering. New knowledge in physics, metallurgy, meteorology, etc., having aeronautical applications will be reported and discussed in the Journal. Papers read at meetings of the Institute will be presented in full or digested for the benefit of members who are unable to attend. It is hoped to include reviews of aeronautical literature and, as soon as practicable, a digest of current aircraft books and magazines will form a section of the publication. The news of the Institute will be furnished members through the Journal. The Journal will be published quarterly.

From time to time, the Institute will, when it appears desirable, provide members with reprints of articles or other publications of interest.

An Annual is published giving the membership list with the constitution and by-laws of the Institute.

Meetings

Meetings of the Institute are held as often as interesting speakers or important aeronautical papers are available. At these meetings, discussion of the subjects presented is invited.

Distinguished foreign aeronautical visitors are received by members at the club rooms at informal gatherings. In this way an opportunity is provided to reciprocate courtesies received by our members when abroad.

An Annual Meeting is held in January of each year at which papers of scientific importance are presented and discussed. A review of progress in the aeronautical sciences during the year is also presented so that members may be kept abreast of current activities.

Membership

The membership of the Institute consists of:

Honorary Fellows	Scientific Members
Honorary Members	Engineering Members
Benefactors	Pilot Members
Fellows	Industrial Members

Technical Members

The following qualifications are used in considering the applications for memberships:

Honorary Fellows of the Institute shall be elected by the Council and shall be persons of pre-eminence in aeronautics.

Honorary Members of the Institute may be elected by the Council for the period of one year.

Benefactors shall be those persons who are interested and active in the advancement of aeronautics and who shall have contributed to the Institute, at one time, the sum of not less than one thousand dollars.

Fellows are elected by the Council from among Members of the Institute of at least six months standing. A Fellow shall have attained a position of distinction and shall have made notable and valuable contributions in one of the aeronautical sciences or aeronautical engineering.

Scientific Members shall be persons of recognized standing in one of the aeronautical sciences.

"Recognized standing" when applied to Scientific Members means that applicants should ordinarily be over thirty years of age. They should have been engaged for a substantial period of time in research, teaching or the application of science to aeronautics. Preferably, this work should not be of an immediate commercial application. Recognized standing is indicated by publications, membership in scientific societies, academic position, record of contributions to knowledge or the like.

Engineering Members shall be persons of recognized standing in aeronautical engineering.

"Recognized standing" when applied to Engineering Members means that applicants should ordinarily be over thirty years of age and, as aeronautical engineers, under general direction or independently, have taken responsibility for important aeronautical engineering work. They should have been in active practice of their profession for at least five years. Alternatively, they should be persons who have been regularly engaged in aeronautical work or closely allied work for at least five years, who have, by invention or proficiency in aeronautical science, the aeronautical arts, or aeronautical literature, attained a standing equivalent to that required of the aeronautical engineers as above described. Standing is indicated by publications, membership in engineering societies, record of aeronautical designs created, and the like.

Pilot Members shall have qualified as pilots and have acquired a recognized standing in the aeronautical field.

"Recognized standing" when applied to Pilot Members means pilots whose work has been primarily related to flight in its scientific or engineering aspects such as test flying. This grade may also include persons of distinction, who are pilots but who would not properly be eligible as Industrial, Engineering or Scientific Members.

Industrial Members shall have been engaged in the application or development of aeronautics and shall have acquired a recognized standing in the aeronautical field. Persons whose work is principally executive or commercial are classed as Industrial Members. Their standing is recognized by their position in the industry.

Technical Members shall be persons who wish to affiliate with the Institute and are engaged in aeronautical engineering, research, teaching, or study, or scientific or engineering work in one of the sciences allied to aeronautics. Pilots and persons engaged in the aeronautical industry or related industries who are interested in the advancement of the aeronautical sciences may be eligible as Technical Members. Persons who may become eligible for other grades of membership upon receipt of more complete information regarding special qualifications may be initially classed as Technical Members.

Graduation from an aeronautical engineering course of a school of recognized standing shall be considered the equivalent of two years aeronautical experience.

Foreign Members—Members, of any grade, not residing in the United States are considered to be foreign correspondents with whom the Institute desires to be associated.

Only Fellows, Scientific Members and Engineering Members shall be entitled to vote by letter ballot or at meetings of the Institute, but Honorary Fellows, Benefactors, Pilot Members and Industrial Members also may serve as officers and on committees.

Abbreviations—The approved abbreviations of the title of the various grades of membership shall be as follows:

Honorary Fellow	Hon. F. I. Ae. S.
Fellows	F. I. Ae. S.
Scientific Members	M. I. Ae. S.
Engineering Members	M. I. Ae. S.

Pilot Members, Industrial Members and Technical Members shall not be entitled to the use of the initials after their names.

Dues

The entrance fee for all members except Honorary Fellows, Honorary Members, Benefactors and Technical Members, shall be five dollars and that for Technical Members three dollars.

The annual dues shall be:

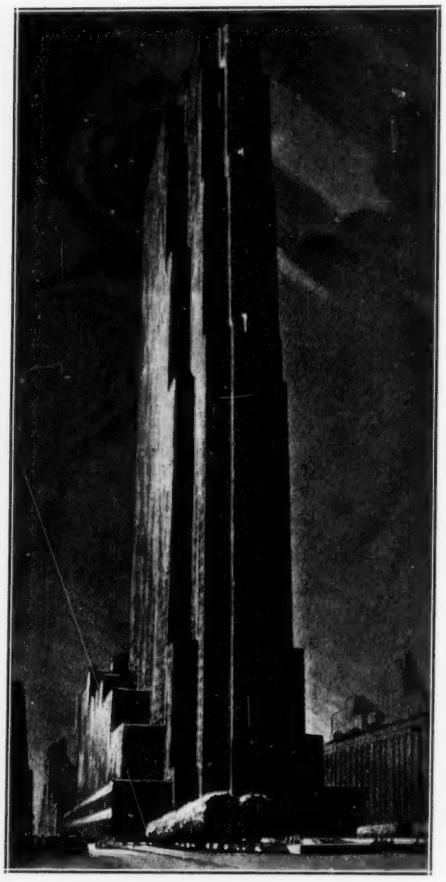
Honorary Fellows	None
Honorary Members	None
Benefactors	None
Fellows	$5.00
Scientific Members	$5.00
Engineering Members	$5.00
Pilot Members	$5.00
Industrial Members	$5.00
Technical Members	$3.00

Application for Membership

Application forms for membership may be secured from any member of the Institute or by requesting them from the Secretary, 5431 RCA Building, Rockefeller Center, New York.

Applicants are required to furnish the names of several persons, preferably members, who are acquainted with their professional work. After these references have been communicated with by the Institute, the Admissions Committee recommends the grade of membership. This recommendation is then passed on by vote of the Council after which an invitation to join may be extended to the applicant.

The Skyport



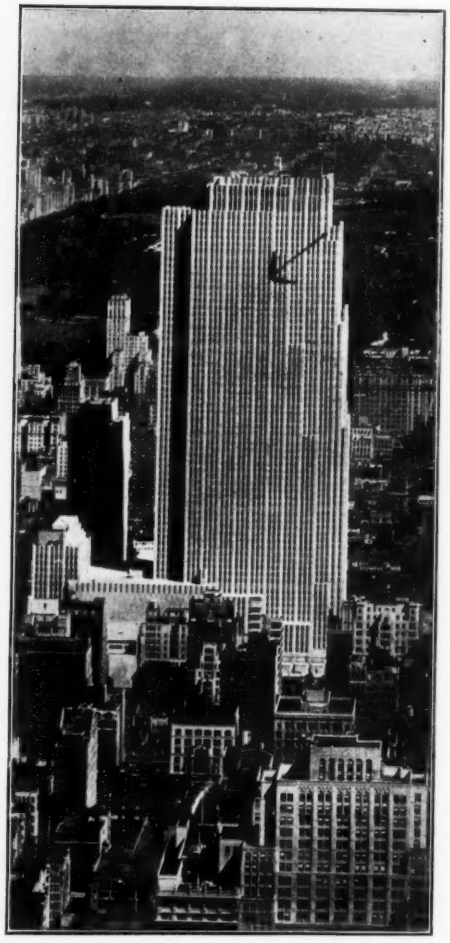
THE R. C. A. BUILDING
ROCKEFELLER CENTER
NEW YORK

THE SKYPORT, *the clubroom and headquarters of the Institute, is located on the fifty-fourth floor of the largest office building in the world. Arrow indicates the location of* THE SKYPORT *at Rockefeller Center.*

The Skyport, the clubroom of the Institute of the Aeronautical Sciences, has been provided for the use of members by generous donors. An anonymous donor has given the rent for the first year and Mr. Grover Loening, a member of the Council, has provided the furnishings. Others have presented books, models, and equipment.

The surroundings of the Skyport may be better understood from the following description of Rockefeller Center:

Rockefeller Center is a permanent century of progress in art, industry and entertainment. Probably the greatest concentration of men and women ever gathered within a single private building development will people Rockefeller Center. When these buildings have been completely tenanted, the total daily population will be more than 215,000. From the RCA Building you can see for a radius of fifty miles. You can



look north to Westchester, east to Long Island, south to the Atlantic and west to New Jersey.

Two international buildings, the British Empire Building and La Maison Francaise, are now complete, and two others, the Palazzo d'Italia and a twin structure, are expected to be shortly in process of construction.

The art program of Rockefeller Center has scope and variety of unusual proportions. Murals, sculpture and other forms of decoration, executed by leading artists, may be found on the exteriors and interiors of all the buildings now completed. Similar ornamentation will be applied to future buildings.

At night the multicolored lights of the city and its suburbs offer a picturesque spectacle. The neon lights of Times Square, the strings of lights in Central Park, the illuminated windows of skyscrapers and the streams of moving headlights on highways and trestles provide an unforgettable panorama.

Necrology

August Goldsmith, an Industrial Member of the I. Ae. S., died in New York City on May 24, 1933. He was born in Birkenfeld, Germany, on May 28, 1860, and came to the United States in 1878. He founded the B. G. Corporation, manufacturers of spark plugs, in 1917 and was its president until his death.

Edward Tracy Birdsall, a Scientific Member of the I. Ae. S., died at his home in New Rochelle, New York, on November 6, 1933, at the age of seventy years. He was one of the founders of the Society of Automotive Engineers and was its first Secretary and Treasurer. He was also a charter member of the American Institute of Electrical Engineers. He graduated as a Mechanical Engineer from the Stevens Institute of Technology in 1886. He was a designer of internal combustion engines since 1900, being one of the pioneers in the automotive field. For several years he had been interested in the development of a Diesel engine for aviation purposes. At the time of his death he was consulting engineer of the Lawrance Engineering and Research Corporation.

Annual Meeting to be held at Columbia University

An all day meeting of the Institute will be held on Wednesday, January 31, 1934, at the Physics Building, Columbia University, Broadway at 120th Street, New York City.

In the morning, commencing at 10 o'clock, the application to aeronautics of Meteorology, Radio, Metallurgy, and Physics will be discussed. A luncheon will be held at 1 P.M. at the Faculty Club of Columbia University for all members and guests. During the afternoon the subjects for discussion will be Aerodynamics, Fuel, Power Plants and Instruments. At five o'clock the Annual Meeting of the Institute will be held. Reports will be made by the officers. New members of the Council will be elected and plans for the year discussed.

In the evening a dinner will be held at the Faculty Club. Some of the earliest workers in aerodynamics, piloting, ballooning, engines and other pioneer fields have been invited to give their earliest impressions of what they were trying to do and to recall early memories.

The Institute was fortunate, last year, in receiving an invitation from Columbia University to hold its Founders' Meeting there. The facilities of the Physics Building and the Faculty Club are so suitable for such a gathering that the Council was pleased to accept the hospitality of the Physics Department again. The Council of the Institute has passed resolutions appreciating the courtesy of Professor George B. Pegram, a scientific member of the Institute and Professor of Physics at Columbia, who has extended the invitation for Columbia and acted as host at our meetings.

It was decided by the Council that, due to the limited time available for the meeting, the presentation and discussion of formal scientific papers would not be advisable. It was thought preferable to have, instead, several of the leading specialists in the fields indicated above to discuss and answer questions on recent knowledge and developments in their particular fields. In this way it is hoped to develop an appraisal of the aeronautical sciences in general. With the assistance of these specialists, the sessions will be conducted by President Hunsaker.

Some of the designs submitted in the competition for a Seal for the Institute. The award was made to a combination of the two seals in the upper left hand corner. See front cover.

The Institute Seal

The Institute needed a seal and the Council authorized a competition for designs which might be suitable. Forty-six different sketches were submitted. There were so many which were excellent that it was difficult to make a choice. Two of the designs seemed to meet the requirements of the competition more nearly than any others. All competitors were given the following instructions:

"It would be well if aeronautical sciences, as distinguished from engineering, could be emphasized. Simplicity and dignity are desired."

The final award for the seal which appears on the cover of this issue was made to Mr. George W. Lewis, Director of Aeronautical Research of the National Advisory Committee for Aeronautics and Mr. W. S. Simpson of the Bureau of Aeronautics, Navy Department. By combining features of both designs, the Council believes that it has secured a seal which is representative of the underlying purpose of the Institute. The Council is appreciative, not only of the designs submitted by the winners of the award, but to all those who submitted other designs. So that members of the Institute can see the various sketches, some are reproduced in this issue.

The Council wishes to thank the following for taking part in this competition. Robert A. Kaiser, B. Kutakoff, M. P. Weihmiller, H. E. Weihmiller, V. Schreckengost, D. J. Leisk, H. McCallister, H. K. Growald, D. E. Dunlap, W. Klemperer, K. M. Daland, G. F. Cummings, H. Case, J. A. Roche, S. T. Payne, J. L. Fechter, R. I. Markey, R. Funk, M. Cozzoli, E. L. Westdal, P. Pevney, J. R. Douglass, K. Huerttle.

Donors to The Skyport

The Institute has received gifts from donors to whom it wishes to express its grateful appreciation.

To the anonymous donor who has provided headquarters and a clubroom at Rockefeller Center for one year the Institute is under great obligation. To the owners of the great building development, Rockefeller Center, Inc., the Institute is indebted for many favors. It is hoped that Rockefeller Center will become the center of aeronautical activity in New York. The various Curtiss-Wright-Sperry corporations and the Manufacturers Aircraft Association have located in the RCA Building.

On October 1st, when the Institute signed its lease it did not own any furnishings. Mr. Grover Loening volunteered to furnish and decorate the office and clubrooms. By this donation, which amounted to over two thousand dollars, Mr. Loening becomes the first Benefactor of the Institute, according to the requirements of the By-Laws.

Other donors to whom the Institute wishes to express its thanks for gifts are as follows:

Aluminum Company of America—Aluminum chairs.

Mr. Luis de Florez—Radio receiver.

Major Lester D. Gardner—Aeronautical library of several hundred volumes, file of aeronautical pamphlets and photographs.

Dr. Henry A. Gardner—Laboratory manual "Paints, Varnishes, Lacquer, and Colors."

Mr. Edward J. Steichen—Photographic mural for the Skyport.

R.K.O. Corporation—Permission to use Mr. Steichen's mural which was originally made for the Center Theater, New York.

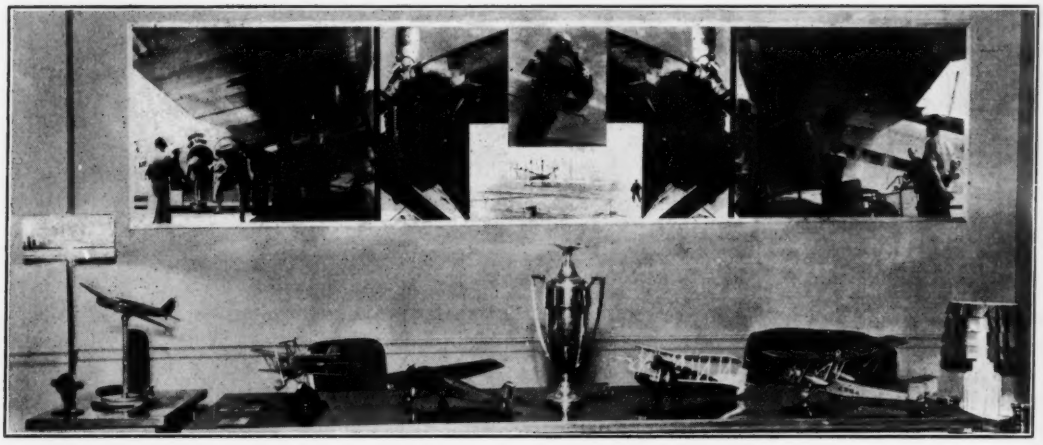
Prof. Alison Reppy—Three volumes of "Air Law Review."

Mr. T. P. Wright—Two Curtiss airplane models.

Mr. R. H. Fleet—Typewriter and airplane model.

Mr. Paul Litchfield—Photographic Mural of airship "Macon."

Members' writing and reading room at THE SKYPORT.



The photographic mural by Edward Steichen showing development of flight. Models on exhibition during week of opening.

Aluminum is used in THE SKYPORT *for furnishings and decorations.*